Sustainable Finance with Matlab

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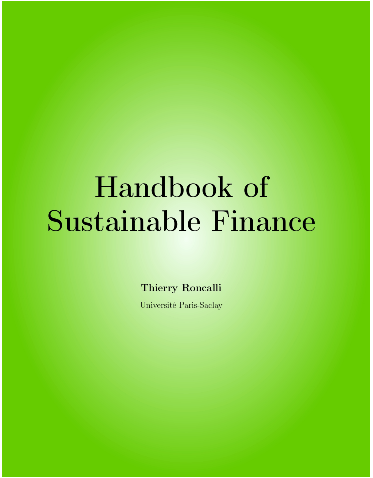
[1]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Handbook of Sustainable Finance (HSF)



Where?

- http://www.thierry-roncalli.com/SustainableFinanceBook.html
- https://ssrn.com/abstract=4277875

What?

- Lectures notes given at the Paris-Saclay University
- 1060 pages, CC BY license (**FREE**)
- LaTeX, PDF and Matlab codes are freely available
- **Matlab programs**
  - 1250 *.m files (3.85 MB)
  - 110 *.mat files (4.25 MB) + external databases ($\approx$ 400 MB, *e.g.*, NGFS, IPCC, WIOD, Exiobase, ERA5, etc.)
  - 300+ figures

Handbook of Sustainable Finance (HSF)

- **Programs of Chapter XX** are located in the directory `root/XX`, for example, all the programs used in Chapter 11 (Climate Portfolio Construction) are in the directory `root/HSF/11. Portfolio Optimization`
- **Functions not specific to sustainable finance** are located in the directory `root/QuantToolbox`, for example, all the functions to perform mean-variance optimization, tracking-error optimization, risk budgeting & parity portfolios, lasso portfolios, Black-Litterman portfolios are in the directory `root/QuantToolbox/rpb`
- **Functions specific to sustainable finance** are located in the directory `root/HSF/0. Toolbox`, for example functions to calculate carbon budgets are located in the directory `root/HSF/0. Toolbox/hsf`

Handbook of Sustainable Finance (HSF, Section 9.3.1, page 750)

Figure: Perfect match between LaTeX code and Matlab code

9.3.1 Carbon budget

Definition

The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period $[t_0, t]$. From a mathematical point of view, it corresponds to the signed area of the region bounded by the function $\mathcal{CE}(t)$:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} \mathcal{CE}(s) \, \mathrm{d}s$$

The carbon budget can be computed with other functions than the carbon emissions. For instance, if the reference level is equal to $\mathcal{CE}^\star(t)$ at time t, we obtain:

$$\mathcal{CB}^\star(t_0, t) = \int_{t_0}^{t} \mathcal{CE}^\star(s) \, \mathrm{d}s$$

Therefore, we can easily compute the excess (or net) carbon budget since we have:

$$\int_{t_0}^{t} \left(\mathcal{CE}(s) - \mathcal{CE}^\star(s)\right) \mathrm{d}s = \mathcal{CB}(t_0, t) - \mathcal{CB}^\star(t_0, t)$$

If the reference level is constant — $\mathcal{CE}^\star(t) = \mathcal{CE}^\star$, the previous formula becomes:

$$\int_{t_0}^{t} \left(\mathcal{CE}(s) - \mathcal{CE}^\star\right) \mathrm{d}s = \mathcal{CB}(t_0, t) - \mathcal{CE}^\star(t - t_0)$$

Example 36 *In Table 9.17, we report the historical data of carbon emissions from 2010 to 2020. Moreover, the company has announced his carbon targets for the years until 2050.*

Table 9.17: Carbon emissions in $\mathrm{MtCO_2e}$

t	2010	2011	2012	2013	2014	2015	2016	2017
$\mathcal{CE}(t)$	4.800	4.950	5.100	5.175	5.175	5.175	5.175	5.100
t	2018	2019	2020	2025*	2030*	2035*	2040*	2050*
$\mathcal{CE}(t)$	5.025	4.950	4.875	4.200	3.300	1.500	0.750	0.150

The asterisk * indicates that the company has announced a carbon target for this year.

Matlab/LaTeX Convention:

- Each example, exercise, table and figure has a **label**
- The label is the **name of the Matlab file** used to perform the calculation or generate the graph

Handbook of Sustainable Finance (HSF, Section 9.3.1, page 750)

Perfect match between LaTeX code and Matlab code

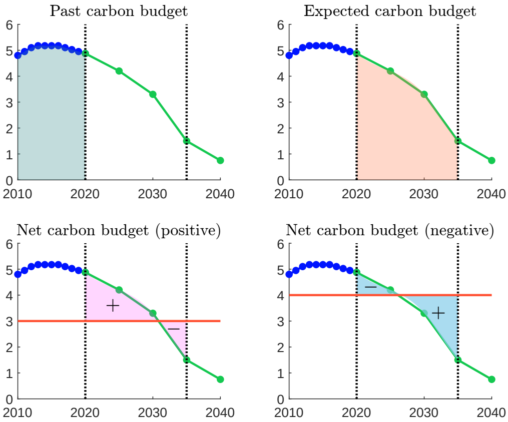
```
\begin{example}
\label{example:chap9-carbon-budget1}
In Table \ref{table:chap9-carbon-budget1}, we report the historical data of carbon emissions from 2010 to 2020.
Moreover, the company has announced his carbon targets for the years until 2050.

\vspace*{-10pt}
\begin{table}[tbph]
\centering
\caption{Carbon emissions in \MtCOtwoEq}
\label{table:chap9-carbon-budget1}
\tableskip
\begin{tabular}{cllllllll}
\hline
$t$                & 2010 &  2011 &  2012 &  2013 &  2014 &  2015 &  2016 &  2017 \\
$\CE\left(t\right)$ & 4.800 & 4.950 & 5.100 & 5.175 & 5.175 & 5.175 & 5.175 & 5.100 \\ \hdashline
$t$                & 2018 &  2019 &  2020 & 2025* & 2030* & 2035* & 2040* & 2050* \\
$\CE\left(t\right)$ & 5.025 & 4.950 & 4.875 & 4.200 & 3.300 & 1.500 & 0.750 & 0.150 \\ \hline
\end{tabular}
\medskip

\noindent
\justifying{{\tablefootsize The asterisk * indicates that the company has announced a carbon target for this year.}}
\end{table}
\end{example}
```

The label is `table:chap9-carbon-budget1` $\Rightarrow$ We deduce that the matlab program is:

`chap9_carbon_budget1.m`

Handbook of Sustainable Finance (chap9_carbon_budget1.m)

Figure: Perfect match between LATEX code and Matlab code



Why Matlab?

Three primary benefits of using MATLAB:

- **Direct mathematical modeling**
  MATLAB's syntax closely resembles mathematical notation, making it straightforward to translate equations into code
- **Enhanced visualization**
  The software's built-in plotting functions facilitate the creation of complex graphics
- **Computational efficiency**
  MATLAB's optimized algorithms and capabilities for handling large/huge datasets enable efficient numerical computations

$\Rightarrow$ **Academic use** (teaching finance) and **professional use** (solving efficient complex problems)

Why Matlab?

Lasso regression

The Lasso regression is a $\mathscr{L}_1$ penalized linear regression:

$$\hat{\beta} = \arg\min \frac{1}{2} (Y - X\beta)^\top (Y - X\beta) + \lambda \|\beta\|_1$$

CCD algorithm for the lasso regression

We have:

$$\beta_j^{(k+1)} = \frac{1}{x_j^\top x_j} \mathscr{S}_\lambda \left(x_j^\top \left(Y - \sum_{j'=1}^{j-1} x_{j'} \beta_{j'}^{(k+1)} - \sum_{j'=j+1}^{m} x_{j'} \beta_{j'}^{(k)} \right) \right)$$

where $\mathscr{S}_\lambda(v)$ is the soft-thresholding operator:
$\mathscr{S}_\lambda(v) = \operatorname{sign}(v) \cdot (|v| - \lambda)_+$

Table: Matlab code

```
for k = 1:nIters
  for j = 1:m
    x_j = X(:,j);
    X_j = X;
    X_j(:,j) = zeros(n,1);
    if lambda > 0
      v = x_j'*(Y - X_j*beta);
      beta(j) = max(abs(v) - lambda,0) * ...
                sign(v) / (x_j'*x_j);
    else
      beta(j) = x_j'*(Y - X_j*beta) / ...
                (x_j'*x_j);
    end
  end
end
```

Why Matlab? (HSF, Section 5.4.1, pages 389-391)

Illustrating the concept of extinction debt (biodiversity risk)

When the remaining habitat area is reduced from A_0 to A, Halley *et al.* (2016) showed that species richness $S(t)$ follows the following dynamics:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \lambda(t) - \frac{k}{n^{\alpha} S_0^{\alpha}} S(t)^{\alpha+1}$$

where $\lambda(t)$ is the origination rate, $n(t) = N(t)/S(t)$ is the average population size per species and ρ is the density of individuals per unit area. If we assume that $\lambda(t) = \lambda$, the equilibrium state $\bar{S}$ is reached when the rate of change in species richness becomes zero:

$$\bar{S} = \left(\frac{\lambda n^{\alpha} S_0^{\alpha}}{k}\right)^{1/(\alpha+1)}$$

This is the value of the steady state after the reduction of the area to A.

Why Matlab? (HSF, Figure 5.39, page 391)

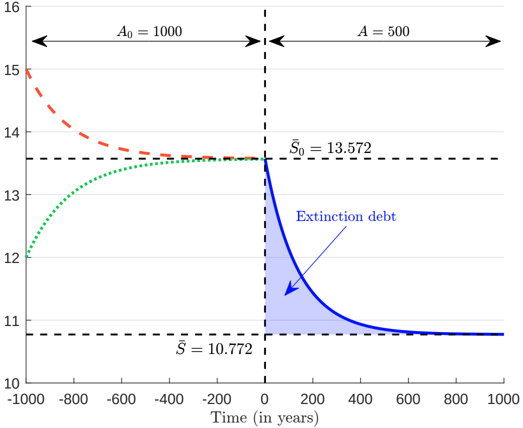
Table: Matlab code (chap5_biodiversity7b.m)

```
A0 = 1000; A = 500; k = 0.10; alpha = [0.25 0.50 0.75]; rho = 10.0; lambda = 0.05;

S0_bar = (lambda .* (rho .* A0).^alpha ./ k).^(1./(alpha+1));
n0 = rho .* A0 ./ S0_bar;
dS0_bar = lambda - k ./ (n0.^alpha .* S0_bar.^alpha) .* S0_bar.^(alpha+1);

S0 = S0_bar;
n = rho.*A./S0;
S_bar = (lambda .* n.^alpha .* S0.^alpha ./ k).^(1./(alpha+1));
dS_bar = lambda - k ./ (n.^alpha .* S0.^alpha) .* S_bar.^(alpha+1);

nIters = size(alpha,2);
t = seqa(0,1,1001);
nt = rows(t);
x_t = zeros(nt,nIters);
y_St = zeros(nt,nIters);

for i = 1:nIters
    dSt = @(t,S) lambda - k./(n(i).^alpha(i) .* S0(i).^alpha(i)) .* S.^(alpha(i)+1);
    [x_t(:,i),y_St(:,i)] = ode45(dSt, t, S0(i));
end
```

Why Matlab? (HSF, Figure 5.39, page 391)

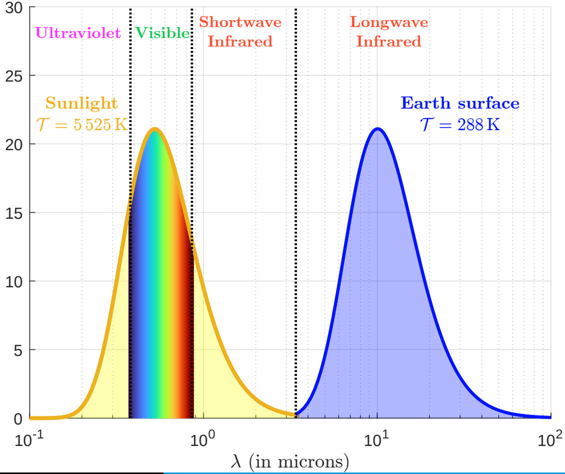
"[...] we illustrate the transition from one steady state to another. We use the following parameters: $A_0 = 1000$, $A = 500$, $k = 0.10$, $\alpha = 0.5$, $\rho = 10$ and $\lambda = 5\%$. Initially, at time $t = -1000$ years, we consider two starting values for species richness: $S(-1000) = 15$ and $S(-1000) = 12$. Both trajectories converge to the steady state value $\bar{S}_0 = 13.572$. At time $t = 0$, we reduce the available habitat by 50%, causing the species richness to shift to a new steady state $\bar{S}_0 = 10.772$. However, it takes time, and the transition between the two equilibria is not instantaneous. It is gradual, resulting in what is known as an extinction debt."

Figure: Extinction debt and steady state



Why Matlab? (HSF, Figure 8.35, page 499)

Figure: Comparison of the radiation spectra of sunlight and the Earth's surface (in $10^{12}\,\mathrm{W/m^2\,m^{-1}}$)



Why Matlab? (HSF, Figure 8.35, page 499)

Table: Matlab code (chap8_physic1c.m)

```
[...]

C = colormap(turbo);
nC = rows(C);
x_min = lambda_UV;
x_max = lambda_VIS;
x = linspace(x_min,x_max,rows(C));
y = planck_law1(x,T1);
for iter = 1:nC
   plot(1e6*[x(iter) x(iter)],[0; y(iter)/1e12],'-','color',C(iter,:),'LineWidth',2.5);
end

[...]

function B_lambda = planck_law1(lambda,T)
c = 2.99792458e8;
h = 6.62608e-34;
k = 1.38066e-23;
w = h * c ./ (k .* T);
B_lambda = (2 * h * c^2) ./ (lambda.^5) ./ (exp(w ./ lambda)-1);
end
```

Why Matlab? (HSF, Figure 8.15, page 475 & Figure 8.65, page 543)

Figure: Gas concentration of Vostok ice cores

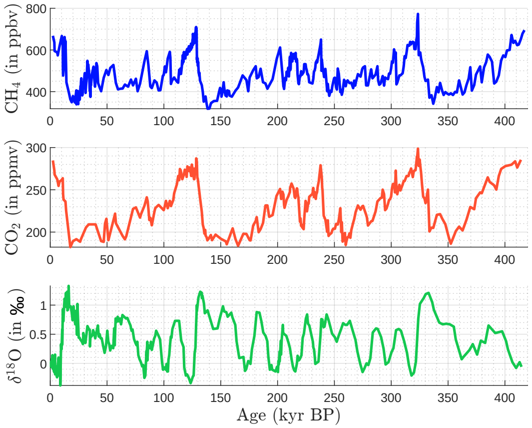
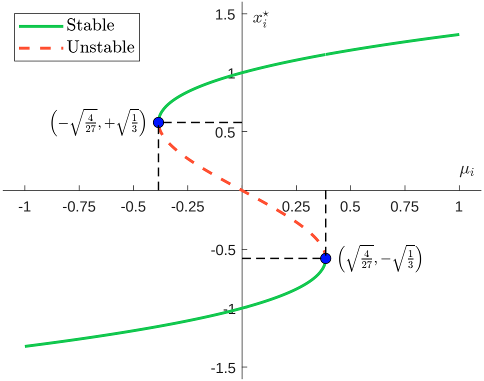


Figure: Double fold bifurcation



Why Matlab? (HSF, Section 80.4, pages 645-704)

- Let A be the input-output matrix of technical coefficients (picture of the supply chain)
- The dimension of A is $nm \times nm$ where n is the number of sector and m is the number of regions
- WIOD: $n = 56$ sectors and $m = 44$ regions $\Rightarrow$ the size of A is 2464×2464 (46 MB)
- Exiobase: $n = 163$ sectors and $m = 44$ regions $\Rightarrow$ the size of A is 7172×7172 (392 MB)

Why Matlab? (HSF, Figure 8.133, page 656 & Figure 8.135, page 657)

Figure: Sparsity pattern of the input-output matrix

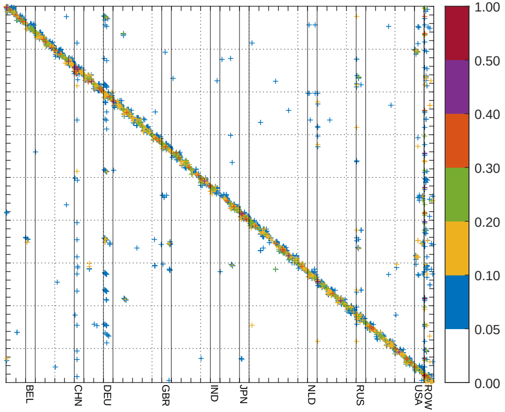
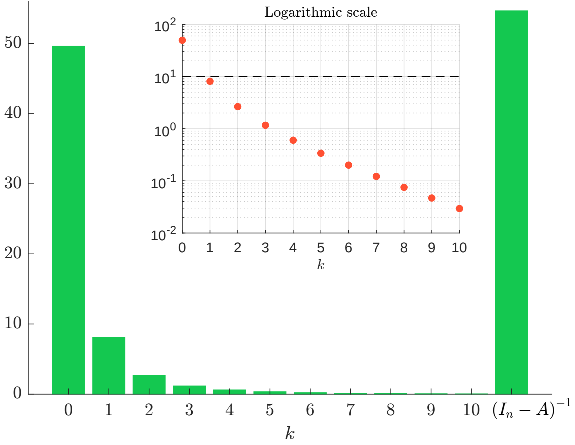


Figure: Frobenious norm of the matrix A^k



Some examples

- ESG scoring (tree-based scoring methods, performance evaluation, backtesting)
- ESG ratings (rating migration matrix, Markov generator)
- Portfolio optimization with ESG scores
- Computing the impact of ESG on the cost-of-capital
- Calculation of the greenium
- Etc.

Persistence of ESG rating systems (HSF, Section 2.3, page 113)

Table: ESG migration matrix #1 (one-year transition probability in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	92.76	5.66	0.90	0.45	0.23	0.00	0.00
AA	4.15	82.73	11.86	0.89	0.30	0.07	0.00
A	0.18	15.47	72.98	10.46	0.82	0.09	0.00
BBB	0.07	1.32	19.60	69.49	9.03	0.42	0.07
BB	0.04	0.19	1.55	19.36	70.88	7.75	0.23
B	0.00	0.05	0.24	1.43	21.54	74.36	2.38
CCC	0.00	0.00	0.22	0.44	2.21	13.24	83.89

Persistence of ESG rating systems (HSF, Figure 2.39, page 132)

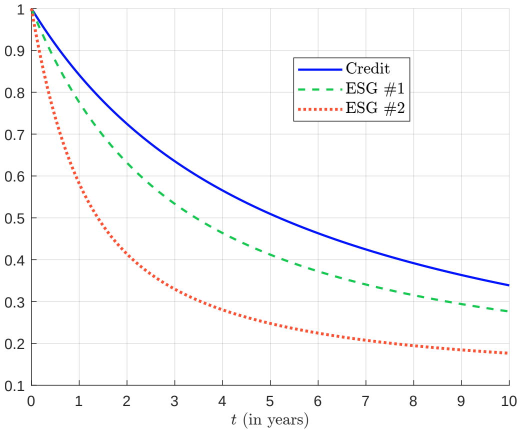
The persistence of a rating system is calculated with the trace statistics:

$$\lambda(t) = \frac{\operatorname{trace}\left(e^{t\Lambda}\right)}{K}$$

where Λ is the markov generator associated to the transition matrix

$\lambda(t)$ is the average probability that states will stay in their states over time (AAA stays AAA, AA stays AA, etc.)

Figure: Trace statistics of credit and ESG migration matrices



Persistence of ESG rating systems (HSF, Figure 2.39, page 132)

Table: Matlab code (chap2_rating_markov13.m)

```
Lambda = logm(P); [Lambda1,Lambda2] = estimate_markov_generator(Lambda);
function [Lambda1,Lambda2] = estimate_markov_generator(Lambda)
    Lambda1 = diagrv(max(Lambda,0),diag(Lambda) + sumc(diagrv(min(Lambda,0),0)'))
    G = abs(diag(Lambda)) + sumc(diagrv(max(Lambda,0),0)');
    B = sumc(diagrv(max(-Lambda,0),0)');
    K = rows(Lambda);
    Lambda2 = Lambda;
    for i = 1:K
        for j = 1:K
            if i ~= j && Lambda(i,j) < 0
                Lambda2(i,j) = 0.0;
            elseif G(i) > 0
                Lambda2(i,j) = Lambda(i,j) - B(i)*abs(Lambda(i,j))/G(i);
            end
        end
    end
end
```

Pedersen-Fitzgibbons-Pomorski model (HSF, Section 3.1.3, page 169)

Model setting

The investment universe consists of n assets. We have $\tilde{R} = R - r \sim \mathcal{N}(\pi, \Sigma)$. The assets have an ESG score given by $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_1, \ldots, \boldsymbol{\mathcal{S}}_n)$. Let $w = (w_1, \ldots, w_n)$ be the investor's portfolio. His initial wealth is W whereas his terminal wealth is given by $\tilde{W} = \left(1 + r + w^\top \tilde{R}\right) W$. The model uses the mean-variance utility function, which is tilted by the ESG score of the portfolio:

$$\boldsymbol{\mathcal{U}}\left(\tilde{W}, w\right) = \mathbb{E}\left[\tilde{W}\right] - \frac{\bar{\gamma}}{2} \operatorname{var}\left(\tilde{W}\right) + \zeta\left(\boldsymbol{\mathcal{S}}(w)\right) W = \left(1 + r + w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta\left(w^\top \boldsymbol{\mathcal{S}}\right)\right) W$$

where ζ is a function that depends on the investor. Optimizing the utility function is equivalent to finding the mean-variance-esg optimized portfolio:

$$\begin{aligned} w^\star &= \arg\max w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta\left(w^\top \boldsymbol{\mathcal{S}}\right) \\ \text{s.t.} &\quad \mathbf{1}_n^\top w = 1 \end{aligned}$$

Let $\sigma(w) = \sqrt{w^\top \Sigma w}$ and $\boldsymbol{\mathcal{S}}(w) = w^\top \boldsymbol{\mathcal{S}}$. The optimization problem can be decomposed as follows:

$$w^\star = \arg\left\{\max_{\bar{\boldsymbol{\mathcal{S}}}}\left\{\max_{\bar{\sigma}}\left\{\max_{w}\left\{f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) \text{ s.t. } w \in \Omega\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right)\right\}\right\}\right\}$$

where $f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) = w^\top \pi - \frac{\bar{\gamma}}{2} \sigma^2(w) + \zeta\left(\boldsymbol{\mathcal{S}}(w)\right)$ and $\Omega = \left\{w \in \mathbb{R}^n : \mathbf{1}_n^\top w = 1, \sigma(w) = \bar{\sigma}, \boldsymbol{\mathcal{S}}(w) = \bar{\boldsymbol{\mathcal{S}}}\right\}$

Pedersen-Fitzgibbons-Pomorski model (HSF, Example 16, page 171)

Example 16

We consider an investment universe with four assets. Their expected returns are 6%, 7%, 8% and 10%, respectively, while their volatilities are 15%, 20%, 25% and 30%. The correlation matrix of the asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & \\ 20\% & 100\% & & \\ 30\% & 50\% & 100\% & \\ 40\% & 60\% & 70\% & 100\% \end{pmatrix}$$

The risk-free rate is set to 2%. The ESG score vector is $\boldsymbol{\mathcal{S}} = (3\%, 2\%, -2\%, -3\%)$.

Pedersen-Fitzgibbons-Pomorski model (HSF, Table 3.12, page 176)

Matlab functions (chap3_pedersen4.m):

- `[w,results,C_x_y] = compute_pedersen_portfolio(mu,r,Sigma,S,sigma_bar,S_bar);`
- `[all_x,mu_x,sigma_x,gamma_x,retcode,lagrange_multipliers] = compute_mvo_portfolio(mu,covMatrix,A,B,C,D,lb,ub,targets,problem,options);`

Table: Optimal portfolios (Example 16)

Statistics	Type-U	Type-A	Type-M					
			$\zeta(s)=s$			$\zeta(s)=0.2\sqrt{\max(s,0)}$		
$\bar{\gamma}$			0.500	1.000	1.500	0.500	1.000	1.500
$\mathcal{S}(w^\star)$	0.017	0.017	0.023	0.028	0.034	0.021	0.024	0.027
$\sigma(w^\star)$	0.139	0.100	0.682	0.329	0.203	0.687	0.339	0.221
$\mathrm{SR}(w^\star \mid r)$	0.345	0.345	0.341	0.329	0.305	0.343	0.339	0.332
$w_1^\star$	0.524	0.378	3.028	1.623	1.090	2.900	1.542	1.072
$w_2^\star$	0.289	0.208	1.786	1.009	0.718	1.673	0.919	0.660
$w_3^\star$	0.120	0.086	0.383	0.073	−0.056	0.464	0.169	0.065
$w_4^\star$	0.067	0.048	−0.012	−0.144	−0.178	0.106	−0.035	−0.079
$w_r^\star$	0.000	0.280	−4.184	−1.562	−0.574	−4.143	−1.596	−0.718

Markowitz optimization with ESG constraints (HSF, Exercise 3.4.1, page 227 & Solution B.1.7, page 934)

Figure: Impact of the minimum ESG score on the efficient frontier (Mean-variance approach)

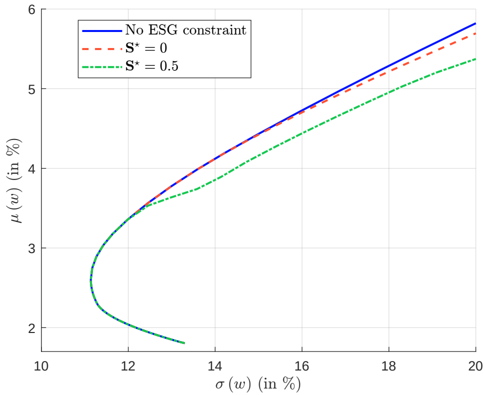
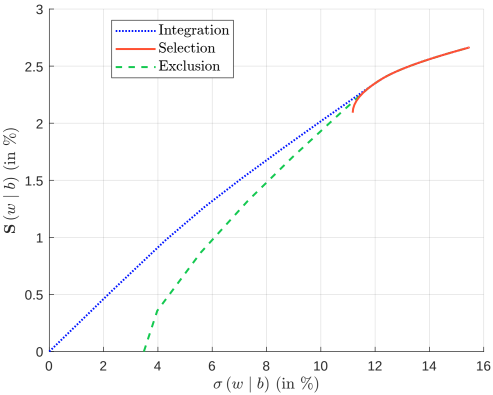


Figure: Impact of ESG strategies on the efficient frontier (Tracking-error approach)



Markowitz optimization with ESG constraints (HSF, Exercise 3.4.1, page 227 & Solution B.1.7, page 934)

Mean-variance optimization

Table: Matlab code (compute_mvo_portfolio)

```
A = ones(1,n); B = 1; C = []; D = [];
lb = zeros(n,1); ub = ones(n,1);
gamma_w = [-1.00:0.01:0 0:0.05:0.95 1.0:0.001:1.50]';
[w1,mu_w1,sigma_w1,gamma_w,retcode] = ...
compute_mvo_portfolio(mu,Sigma,A,B,C,D,lb,ub,gamma_w,0);
esg_w1 = w1'*esg;

C = -esg'; D = 0;
[w2,mu_w2,sigma_w2,gamma_w,retcode] = ...
compute_mvo_portfolio(mu,Sigma,A,B,C,D,lb,ub,gamma_w,0);
esg_w2 = w2'*esg;

D = -0.5;
[w3,mu_w3,sigma_w3,gamma_w,retcode] = ...
compute_mvo_portfolio(mu,Sigma,A,B,C,D,lb,ub,gamma_w,0);
esg_w3 = w3'*esg;
```

Tracking-error variance optimization

Table: Matlab code (compute_te_portfolio)

```
A = ones(1,n); B = 1; C = []; D = [];
lb = zeros(n,1); ub = ones(n,1);
gamma_w = [0:0.001:0.05]';
[w1,esg_w1,sigma_w1,gamma_w,retcode] = ...
compute_te_portfolio(b,esg,Sigma,A,B,C,D,lb,ub,gamma_w,0);

ub = ones(n,1); ub(4:6) = zeros(3,1);
[w2,esg_w2,sigma_w2,gamma_w,retcode] = ...
compute_te_portfolio(b,esg,Sigma,A,B,C,D,lb,ub,gamma_w,0);

ub = ones(n,1); ub(6) = 0;
[w3,esg_w3,sigma_w3,gamma_w,retcode] = ...
compute_te_portfolio(b,esg,Sigma,A,B,C,D,lb,ub,gamma_w,0);
```

Matlab file: `chap16_chap3_exercise1.m`

Some examples

- Computing climate sensitivity and feedback
- Bifurcation theory and tipping points
- Simulation of the DICE model
- Solving environmentally-extended input-output models
- Calculating global warming potential (GWP)
- Carbon intensity of investment portfolios
- Carbon budget, carbon trend & carbon momentum
- Bond and equity optimized portfolios with climate measures
- Decarbonized and net-zero investment portfolios
- Etc.

Calculating CO_2e (HSF, Section 9.1.1, page 706)

- The mathematical definition of the global warming potential is:

$$\mathrm{gwp}_i(t) = \frac{\mathrm{Agwp}_i(t)}{\mathrm{Agwp}_0(t)} = \frac{\int_0^t RF_i(s)\,\mathrm{d}s}{\int_0^t RF_0(s)\,\mathrm{d}s} = \frac{\int_0^t A_i(s)\,\mathbf{S}_i(s)\,\mathrm{d}s}{\int_0^t A_0(s)\,\mathbf{S}_0(s)\,\mathrm{d}s}$$

  where $A_i(t)$ is the radiative efficiency value of gas i, $\mathbf{S}_i(t)$ is the decay function and $i = 0$ is the reference gas (e.g, CO_2)

- For the carbon dioxide gas, we have $A_{\mathrm{CO_2}} = 1.76 \times 10^{-18}$ and:

$$\mathbf{S}_{\mathrm{CO_2}}(t) = 0.2173 + 0.2240 \cdot \exp\left(-\frac{t}{394.4}\right) + 0.2824 \cdot \exp\left(-\frac{t}{36.54}\right) + 0.2763 \cdot \exp\left(-\frac{t}{4.304}\right)$$

- For the methane gas, we have $A_{\mathrm{CH_4}} = 2.11 \times 10^{-16}$ and:

$$\mathbf{S}_{\mathrm{CH_4}}(t) = \exp\left(-\frac{t}{12.4}\right)$$

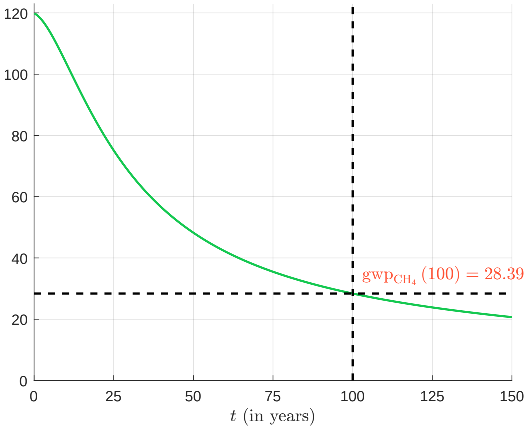
Why 1 kg of $CH_4 = 28$ kg of CO_2? (HSF, Figure 9.4, page 711)

- The instantaneous global warming potential of the methane is equal to:

$$\mathrm{gwp}_{\mathrm{CH}_4}(0) = \frac{A_{\mathrm{CH}_4}}{A_{\mathrm{CO}_2}} = \frac{2.11 \times 10^{-16}}{1.76 \times 10^{-18}} \approx 119.9$$

- After 100 years, we obtain $\mathrm{gwp}_{\mathrm{CH}_4}(100) = 28.3853$, which is the value calculated by IPCC (2013)
- 1 kg of $\mathrm{CH}_4 \approx 28$ kg of CO_2

Figure: Global warming potential for methane



Calculating CO_2e (HSF, Figure 9.4, page 711)

Table: Matlab code (chap9_gwp2d.m)

```
decay_CO2 = @(t) 0.2173 + 0.2240 * exp(-t/394.4) + 0.2824 * exp(-t/36.54) + 0.2763 * exp(-t/4.304);
decay_CH4 = @(t) exp(-t/12.4);

A_CO2 = 1.76; A_CH4 = 2.11;
RF_CO2 = @(t) A_CO2 * decay_CO2(t);
RF_CH4 = @(t) A_CH4 * decay_CH4(t);
AGWP_CO2 = @(t) integral(RF_CO2,0,t);
AGWP_CH4 = @(t) integral(RF_CH4,0,t);

t = [seqa(0,0.1,10); seqa(1,1,150)]; n = rows(t); y1 = zeros(n,1); y2 = zeros(n,1);
for iter = 1:n
    y1(iter) = AGWP_CO2(t(iter));
    y2(iter) = AGWP_CH4(t(iter));
end

gwp = ((1e-16)*y2) ./ (1e-18*y1);
gwp_100 = 100*AGWP_CH4(100)/AGWP_CO2(100);
```

Input-output analysis of carbon emissions (HSF, Section 8.4.1, page 645)

- The input-output (IO) model was introduced by Leontief to quantify the interdependencies between different sectors in a single or multi-regional economy

- n different sectors, $Z_{i,j}$ is the value of transactions from sector i to sector j, y_i is the final demand for products sold by sector i, x_i is the total production of sector i

$$\underbrace{x_i}_{\text{Supply}} = \underbrace{\sum_{j=1}^{n} Z_{i,j} + y_i}_{\text{Demand}} \qquad \text{or} \qquad x = Ax + y$$

  where $A = (A_{i,j}) = Z \operatorname{diag}(x)^{-1}$ is the input-output matrix of the technical coefficients

- Assuming that final demand is exogenous, technical coefficients are fixed, we obtain:

$$x = (I_n - A)^{-1} y$$

- $\mathcal{L} = (I_n - A)^{-1}$ is known as the Leontief inverse (or multiplier) matrix and represents the amount of total output from sector i that is required by sector j to satisfy its final demand

Input-output analysis of carbon emissions (HSF, Section 8.4.3, page 661)

Estimation of first-tier and indirect emissions

Let A be the matrix of technical coefficients. We have:

$$\mathcal{CI}_{\text{total}} = \mathcal{L}^\top \mathcal{CI}_1 = \left(I_n - A^\top\right)^{-1} \mathcal{CI}_1$$

It follows that the indirect carbon intensities are given by:

$$\mathcal{CI}_{\text{indirect}} = \mathcal{CI}_{\text{total}} - \mathcal{CI}_1 = \left(\left(I_n - A^\top\right)^{-1} - I_n\right) \mathcal{CI}_{\text{direct}}$$

In particular, we can decompose $\mathcal{CI}_{\text{indirect}}$ using the Neumann series:

$$\mathcal{CI}_{\text{indirect}} = \underbrace{A^\top \mathcal{CI}_1}_{\text{First-tier}} + \underbrace{\left(A^\top\right)^2 \mathcal{CI}_1}_{\text{Second-tier}} + \ldots + \underbrace{\left(A^\top\right)^k \mathcal{CI}_1}_{k^{th}\text{-tier}} + \ldots$$

Input-output analysis of carbon emissions (HSF, Example 26, page 648)

Example 26

We consider the following basic economy:

		To				Final Demand	Total Output
		Energy	Materials	Industrials	Services		
		Z				y	x
From	Energy	500	800	1 600	1 250	850	5 000
	Materials	500	400	1 600	625	875	4 000
	Industrials	250	800	2 400	1 250	3 300	8 000
	Services	100	200	800	4 375	7 025	12 500
Value added	Labour	3 000	800	1 000	3 000		
	Capital	650	1 000	600	2 000		
	Income	5 000	4 000	8 000	12 500		

Input-output analysis of carbon emissions (HSF, Table 8.46, page 662 & Table 8.47 page 663)

Table: Direct and indirect carbon intensities

Sector	$\mathcal{CI}_1$	$\mathcal{CI}_{\text{total}}$	$\mathcal{CI}_{\text{direct}}$	$\mathcal{CI}_{\text{indirect}}$	$\mathcal{CI}_{\text{direct}}$	$\mathcal{CI}_{\text{indirect}}$	$\dfrac{\mathcal{CI}_{\text{total}}}{\mathcal{CI}_1}$
	(in tCO_2e/\$ mn)				(in %)		
Energy	100.00	131.49	100.00	31.49	76.05%	23.95%	1.31
Materials	50.00	113.69	50.00	63.69	43.98%	56.02%	2.27
Industrials	25.00	114.62	25.00	89.62	21.81%	78.19%	4.58
Services	10.00	61.99	10.00	51.99	16.13%	83.87%	6.20

Table: Tier decomposition of carbon intensities

	Sector	1	2	3	4	5	10	15	∞
$\mathcal{CI}_{(k)}$	Energy	16.45	6.99	3.60	1.97	1.09	0.06	0.00	0.00
	Materials	30.50	14.97	8.13	4.47	2.48	0.14	0.01	0.00
	Industrials	38.50	22.79	12.58	6.96	3.88	0.21	0.01	0.00
	Services	18.50	13.50	8.45	4.98	2.86	0.16	0.01	0.00

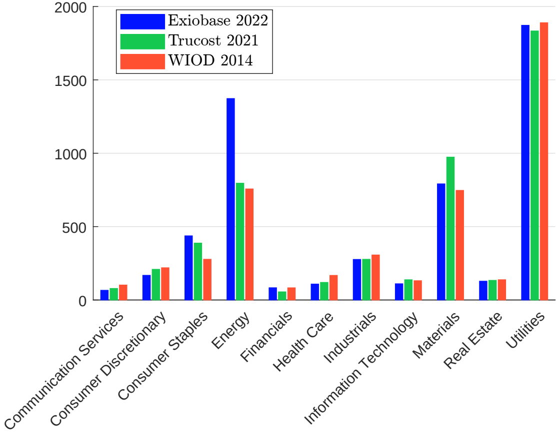
Input-output analysis of carbon emissions (HSF, Table 8.46, page 662)

Table: Matlab code (chap8_eeio_iot6a.m)

```
A = [0.10 0.20 0.20 0.10;
     0.10 0.10 0.20 0.05;
     0.05 0.20 0.30 0.10;
     0.02 0.05 0.10 0.35];
CE_1 = 1000*[500; 200; 200; 125];
x = [5000; 4000; 8000; 12500];
CI_1 = CE_1 ./ x;
[CI_direct,CI_indirect,CI_123,CE_direct,CE_indirect,CE_123,eeio_results] = eeio_compute_impact2(A,CI_1);

function [CI_direct,CI_indirect,CI_123,CE_direct,CE_indirect,CE_123,results] = ...
                                                  eeio_compute_impact2(A,CI_1,CE_1,K)
n = rows(A); I = eye(n); L = inv(I - A');

CI_123 = L * CI_1; CI_direct = CI_1; CI_indirect = CI_123 - CI_1;

[...]
end
```

Input-output analysis of carbon emissions (HSF, Figure 8.140, page 674)

Figure: Total carbon intensity $\mathcal{CI}_{\text{total}}$ by GICS sector (MSCI World Index, May 2023)



Taxation & pass-through (HSF, Section 8.4.5, page 692)

Let $\phi = (\phi_1, \ldots, \phi_n)$ and $\Phi = \operatorname{diag}(\phi)$ be the pass-through vector and matrix. We have:

$$\Delta p = \sum_{k=0}^{\infty} \left(A^\top \Phi\right)^k \Phi \Delta \upsilon = \left(I_n - A^\top \Phi\right)^{-1} \Phi \Delta \upsilon = \tilde{\mathcal{L}}(\phi) \Delta \upsilon$$

where $\tilde{\mathcal{L}}(\phi) = \left(I_n - A^\top \Phi\right)^{-1} \Phi$

Applying the previous analysis to the carbon tax, we have $\Delta \upsilon = t_{\mathrm{direct}}$. We deduce that:

$$\left\{\begin{array}{l}
T_{\mathrm{producer}} = x \odot (I_n - \Phi)\, t_{\mathrm{direct}} = x \odot (\mathbf{1}_n - \phi) \odot t_{\mathrm{direct}} = (\mathbf{1}_n - \phi) \odot T_{\mathrm{direct}} \\
T_{\mathrm{consumer}} = T_{\mathrm{downstream}} = x \odot \tilde{\mathcal{L}}(\phi)\, t_{\mathrm{direct}} \\
T_{\mathrm{total}} = T_{\mathrm{producer}} + T_{\mathrm{consumer}} = x \odot \left(I_n - \Phi + \tilde{\mathcal{L}}(\phi)\right) t_{\mathrm{direct}} \\
T_{\mathrm{direct}} = x \odot t_{\mathrm{direct}} \\
T_{\mathrm{indirect}} = T_{\mathrm{total}} - T_{\mathrm{direct}} = x \odot \left(\tilde{\mathcal{L}}(\phi) - \operatorname{diag}(\phi)\right) t_{\mathrm{direct}} \\
R_{\mathrm{government}} = T_{\mathrm{direct}} = x \odot t_{\mathrm{direct}}
\end{array}\right.$$

Taxation & pass-through (HSF, Figures 8.147 and 8.148, page 693)

Figure: Producer and consumer cost contributions (uniform pass-through rate)

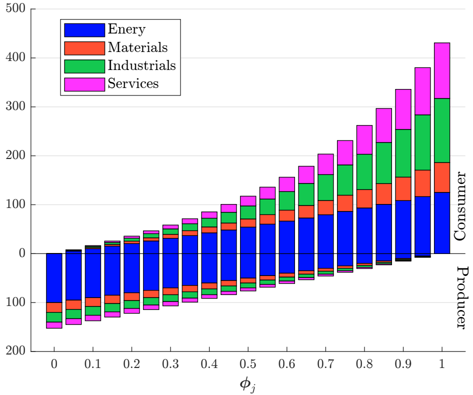
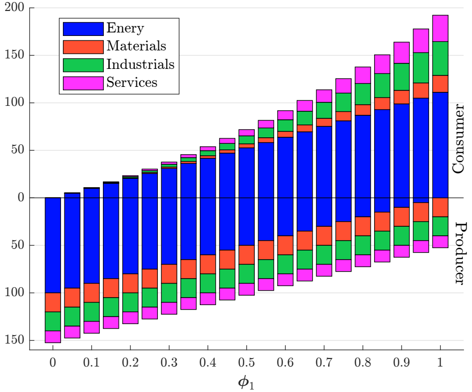


Figure: Producer and consumer cost contributions ($\phi_2 = \phi_3 = \phi_4 = 0\%$)



Input-output analysis of carbon emissions (HSF, Figure 8.147, page 693)

Table: Matlab code (chap8_eeio_price4.m)

```
A = [0.10 0.20 0.20 0.10;
     0.10 0.10 0.20 0.05;
     0.05 0.20 0.30 0.10;
     0.02 0.05 0.10 0.35];
CI1 = [100; 50; 25; 10];
V = [3650; 1800; 1600;  5000];
x = [5000; 4000; 8000; 12500];
CE1 = x .* CI1;

tax = [200; 100; 100; 100]/1e6;
phi = seqa(0,0.05,21);
alpha = [0.10; 0.20; 0.30; 0.40];
[Delta_p,T_direct,T_total,results] = eeio_carbon_tax(A,x,V,CE1,CI1,tax,alpha,phi);

T_producer = results.T_producer;
T_consumer = results.T_consumer;

function [Delta_p,T_direct,T_total,results] = eeio_carbon_tax(A,x,V,CE1,CI1,tau,alpha,phi)
[...]
end
```

Economic impact of a global carbon tax (HSF, Table 8.68, page 701)

Table: Global carbon tax impact on five most and least affected countries ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Region	Cost						Revenue
	$\mathcal{C}ost_{\text{total}}$	$\mathcal{C}ost_{\text{direct}}$	$\mathcal{C}ost_{\text{indirect}}$	$\mathcal{C}ost_{\text{producer}}$	$\mathcal{C}ost_{\text{downstream}}$	$\mathcal{C}ost_{\text{net}}$	$\mathcal{R}_{\text{government}}$
World	5.01%	2.82%	2.18%	0.93%	4.08%	2.18%	2.82%
RUS	12.79%	8.55%	4.24%	1.44%	11.34%	4.24%	8.55%
IND	11.38%	6.83%	4.55%	2.28%	9.11%	4.55%	6.83%
IDN	7.85%	5.53%	2.31%	2.08%	5.77%	2.31%	5.53%
CHN	7.47%	3.44%	4.03%	1.21%	6.26%	4.03%	3.44%
BGR	7.07%	3.94%	3.12%	0.89%	6.18%	3.12%	3.94%
DNK	1.47%	0.98%	0.49%	0.54%	0.93%	0.49%	0.98%
FRA	1.39%	0.79%	0.60%	0.35%	1.04%	0.60%	0.79%
SWE	1.21%	0.59%	0.62%	0.21%	1.00%	0.62%	0.59%
LUX	1.15%	0.51%	0.64%	0.35%	0.80%	0.64%	0.51%
CHE	0.75%	0.30%	0.45%	0.16%	0.59%	0.45%	0.30%

$\Rightarrow$ **Only 20% of the costs are borne by producers**

Economic impact of a European carbon tax (HSF, Table 8.69, page 702)

Table: Economic impact of a EU carbon tax ($100/tCO$_2$e, Exiobase 2022)

Region	Cost						Revenue
	$\mathscr{C}ost_{\text{total}}$	$\mathscr{C}ost_{\text{direct}}$	$\mathscr{C}ost_{\text{indirect}}$	$\mathscr{C}ost_{\text{producer}}$	$\mathscr{C}ost_{\text{downstream}}$	$\mathscr{C}ost_{\text{net}}$	$\mathscr{R}_{\text{government}}$
World	0.36%	0.22%	0.14%	0.07%	0.28%	0.14%	0.22%
BGR	6.30%	3.94%	2.35%	0.89%	5.41%	2.35%	3.94%
GRC	5.64%	4.61%	1.03%	2.52%	3.12%	1.03%	4.61%
POL	5.21%	3.44%	1.77%	0.98%	4.24%	1.77%	3.44%
CYP	4.86%	3.94%	0.92%	2.49%	2.37%	0.92%	3.94%
CZE	3.90%	2.13%	1.76%	0.44%	3.46%	1.76%	2.13%
ROU	3.60%	2.19%	1.41%	0.69%	2.91%	1.41%	2.19%
PRT	3.28%	2.13%	1.15%	0.70%	2.58%	1.15%	2.13%
LTU	3.22%	2.41%	0.82%	1.00%	2.22%	0.82%	2.41%
LVA	3.11%	2.15%	0.96%	1.07%	2.05%	0.96%	2.15%
HRV	2.88%	2.18%	0.70%	0.89%	1.99%	0.70%	2.18%
SVK	2.72%	1.62%	1.09%	0.42%	2.30%	1.09%	1.62%
HUN	2.70%	1.83%	0.87%	0.61%	2.08%	0.87%	1.83%
SVN	2.38%	1.51%	0.87%	0.47%	1.91%	0.87%	1.51%
FIN	2.27%	1.36%	0.91%	0.36%	1.91%	0.91%	1.36%
ESP	1.82%	1.15%	0.68%	0.41%	1.41%	0.68%	1.15%

$\Rightarrow$ **95% of the costs fall on European countries**

PPI impact of a carbon tax (HSF, Table 8.72, page 704)

Table: Producer price index (π_{ppi}) estimates ($\$100/\text{tCO}_2\text{e}$, Exiobase 2022)

Rank	Global tax		EU tax		US tax		China tax	
	World	4.08%	World	0.28%	World	0.37%	World	1.38%
1	RUS	11.34%	BGR	5.41%	USA	1.62%	CHN	5.68%
2	IND	9.11%	POL	4.24%	CAN	0.18%	ROW	0.13%
3	CHN	6.26%	CZE	3.46%	MEX	0.18%	KOR	0.12%
4	BGR	6.18%	GRC	3.12%	KOR	0.07%	MEX	0.06%
5	IDN	5.77%	ROU	2.91%	IRL	0.06%	IND	0.05%
6	ROW	5.68%	PRT	2.58%	BRA	0.05%	IDN	0.05%
7	POL	4.86%	CYP	2.37%	TWN	0.04%	JPN	0.04%
8	MEX	4.57%	SVK	2.30%	ROW	0.04%	POL	0.04%
9	TWN	4.41%	LTU	2.22%	IND	0.04%	CZE	0.04%
10	TUR	4.39%	HUN	2.08%	NLD	0.03%	HUN	0.04%
11	CZE	4.03%	LVA	2.05%	GBR	0.03%	TUR	0.04%
12	GRC	3.87%	HRV	1.99%	NOR	0.02%	CAN	0.03%
13	KOR	3.85%	SVN	1.91%	BEL	0.02%	BEL	0.03%
14	AUS	3.82%	FIN	1.91%	JPN	0.02%	SVK	0.03%
15	ROU	3.42%	AUT	1.50%	CHN	0.02%	AUS	0.03%

Results on PPI

- Producer inflation: 4.08%
- Emerging markets are the most affected
- Regional taxation penalized the domestic economy

CPI impact of a carbon tax (HSF, Table 8.73, page 704)

Table: Consumer price index (π_{cpi}) estimates ($\$100/\text{tCO}_2\text{e}$, Exiobase 2022)

Rank	Global tax		EU tax		US tax		China tax	
	World	3.53%	World	0.48%	World	0.27%	World	1.15%
1	IDN	6.75%	FRA	5.95%	USA	1.06%	CHN	5.88%
2	CHN	6.35%	CZE	4.07%	MEX	0.16%	ROW	0.16%
3	FRA	6.29%	HRV	3.83%	CAN	0.16%	KOR	0.08%
4	IND	5.98%	GRC	3.59%	IRL	0.05%	AUS	0.07%
5	RUS	5.72%	POL	3.49%	BRA	0.04%	IND	0.07%
6	CZE	4.63%	CYP	3.32%	GBR	0.04%	CAN	0.07%
7	HRV	4.42%	BGR	3.16%	ROW	0.04%	MEX	0.06%
8	GRC	4.35%	SVK	2.80%	KOR	0.03%	TUR	0.05%
9	POL	4.14%	MLT	2.69%	IND	0.03%	IDN	0.04%
10	BGR	3.89%	PRT	2.58%	NOR	0.03%	BRA	0.04%
11	ROW	3.82%	LUX	2.30%	NLD	0.03%	JPN	0.04%
12	TWN	3.73%	HUN	2.20%	LUX	0.03%	BEL	0.04%
13	CYP	3.57%	LTU	2.11%	TWN	0.02%	RUS	0.04%
14	MLT	3.38%	NLD	2.11%	BEL	0.02%	GRC	0.04%
15	SVK	3.36%	SVN	1.90%	TUR	0.02%	POL	0.04%

Results on CPI

- Consumer inflation: 3.53%
- Consumption allocation $\neq$ production allocation
- A Chinese carbon tax puts relatively more pressure on the global value chain than global consumption

Portfolio decarbonization (HSF, Section 11.2, page 803)

Equity portfolios

$$
\begin{aligned}
w^\star &= \arg\min \frac{1}{2}(w-b)^\top \Sigma (w-b) \\
\text{s.t.} & \left\{ \begin{array}{l} \mathcal{CI}(w) \leq (1-\mathcal{R})\,\mathcal{CI}(b) \\ w \in \Omega_0 \cap \Omega \end{array} \right.
\end{aligned}
$$

Corporate bond portfolios

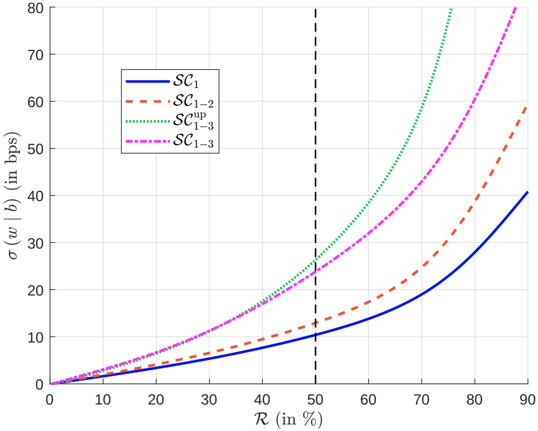
$$
\begin{aligned}
w^\star &= \arg\min \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| + \\
& \quad \lambda \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\,\mathrm{DTS}_i \right| \\
\text{s.t.} & \left\{ \begin{array}{l} \mathcal{CI}(w) \leq (1-\mathcal{R})\,\mathcal{CI}(b) \\ w \in \mathscr{C}_0 \cap \mathscr{C}'_1 \cap \mathscr{C}'_3 \cap \mathscr{C}'_4 \end{array} \right.
\end{aligned}
$$

Tracking-error variance
Quadratic programming: **quadprog**

Active risk (active share + DTS + MD)
Linear programming: **linprog**

Equity portfolio decarbonization (HSF, Figure 11.7, page 818)

Figure: Impact of the carbon scope on the tracking error volatility (MSCI World, June 2022, $\mathscr{C}_0$ constraint)



Equity portfolio decarbonization (HSF, Table 11.15, page 818)

Table: Sector allocation in % (MSCI World, June 2022, $\mathscr{C}_0$ constraint, scope $\mathcal{SC}_{1-3}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.15	8.42	8.78	9.34	10.13	12.27
Consumer Discretionary	10.56	10.69	10.69	10.65	10.52	10.23	9.62	6.74
Consumer Staples	7.80	7.80	7.69	7.48	7.11	6.35	5.03	1.77
Energy	**4.99**	4.14	3.65	3.10	2.45	**1.50**	**0.49**	**0.00**
Financials	**13.56**	14.53	15.17	15.94	16.90	**18.39**	**20.55**	**28.62**
Health Care	14.15	14.74	15.09	15.50	16.00	16.78	17.77	17.69
Industrials	9.90	9.28	9.01	8.71	8.36	7.79	7.21	6.03
Information Technology	21.08	21.68	22.03	22.39	22.88	23.51	24.12	24.02
Materials	**4.28**	3.78	3.46	3.06	2.56	**1.85**	**1.14**	**0.24**
Real Estate	2.90	3.12	3.27	3.41	3.57	3.72	3.71	2.51
Utilities	**3.21**	2.28	1.79	1.36	0.90	**0.54**	**0.24**	**0.12**

Strategy **long on Financials** and **short on Energy, Materials and Utilities**

Bond portfolio decarbonization (HSF, Figures 11.15 and 11.16, page 826)

Figure: Impact of the carbon scope on the active share in % (ICE Global Corp., June 2022)

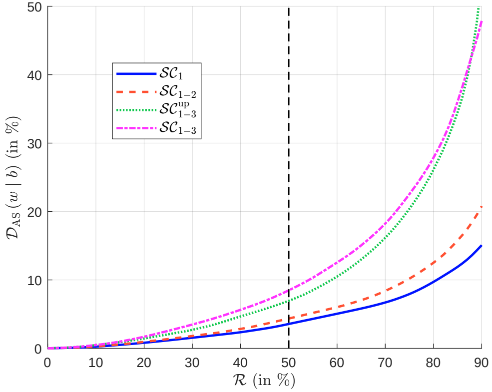
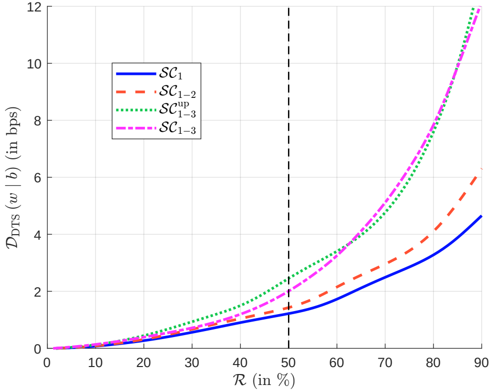


Figure: Impact of the carbon scope on the DTS risk in bps (ICE Global Corp., June 2022)



Bond portfolio decarbonization (HSF, Table 11.18, page 825)

Table: Sector allocation in % (ICE Global Corp., June 2022, scope $\mathcal{SC}_{1-3}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	7.35	7.34	7.37	7.43	7.43	7.31	7.30
Consumer Discretionary	5.97	5.97	5.96	5.94	5.93	5.46	4.48	3.55
Consumer Staples	6.04	6.04	6.04	6.04	6.04	6.02	5.39	4.06
Energy	**6.49**	5.49	4.42	3.84	3.69	**3.23**	**2.58**	**2.52**
Financials	**33.91**	34.64	35.66	35.96	36.09	**37.36**	**38.86**	**39.00**
Health Care	7.50	7.50	7.50	7.50	7.50	7.50	7.52	7.48
Industrials	**8.92**	9.38	9.62	10.19	11.34	**12.07**	**13.55**	**18.13**
Information Technology	5.57	5.57	5.59	5.59	5.60	5.60	5.52	5.27
Materials	3.44	3.43	3.31	3.18	3.12	2.64	2.25	1.86
Real Estate	4.76	4.74	4.74	4.74	4.74	4.66	4.61	3.93
Utilities	**10.06**	9.89	9.82	9.64	8.52	**8.04**	**7.92**	**6.88**

Strategy **long on Financials and Industrials** and **short on Energy, Materials and Utilities**

Net-zero investing (HSF, Section 11.3, pages 827-865)

Two main approaches

1. Integrated approach (complex top-down portfolio optimization)
2. Core-satellite approach (bottom-up portfolio allocation)

Integrated approach

- Equity and bond mutual funds
- ETFs
- Indexes

Core-satellite approach

- Multi-asset portfolios
- Thematic investment
- Strategic asset allocation

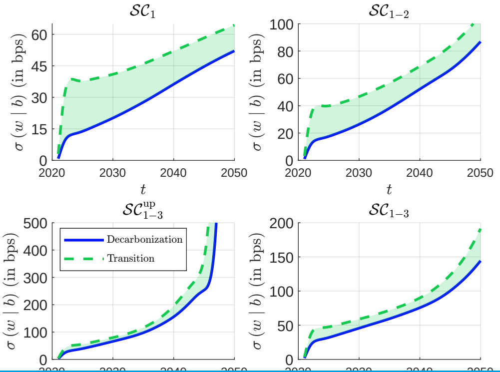
Net-zero investing (HSF, Figure 11.22, page 845)

We solve the following optimization problem:

$$
\begin{aligned}
w^{\star}(t) &= \arg\min \frac{1}{2}\left(w-b(t)\right)^{\top}\Sigma(t)\left(w-b(t)\right) \\
&\text{s.t.} \quad \begin{cases}
\mathcal{CI}(t,w) \leq (1-\mathcal{R}(t_0,t))\,\mathcal{CI}(t_0,b(t_0)) \\
\mathcal{CM}(t,w) \leq \mathcal{CM}^{\star}(t) \\
\mathcal{GI}(t,w) \geq (1+\mathcal{G})\,\mathcal{GI}(t,b(t)) \\
w \in \mathscr{C}_0 \cap \mathscr{C}_3(0,10,2)
\end{cases}
\end{aligned}
$$

where $\mathcal{CI}(t,w)$ is the portfolio carbon intensity, $\mathcal{CM}(t,w)$ is the portfolio carbon momentum, $\mathcal{GI}(t,w)$ is the portfolio green intensity, $\mathcal{CM}^{\star}(t) = -5\%$ and $\mathcal{G} = 100\%$

Figure: Tracking error volatility of net-zero portfolios (MSCI World, June 2022, $\mathscr{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)



Net-zero investing (HSF, Section 11.3.2, page 853)

The core-satellite approach

Decarbonized portfolio

- Carbon intensity
- Decarbonization pathway(s)
- Top-down approach
- Portfolio construction
- Net zero **carbon** metrics

$+$

Transition portfolio

- Green intensity
- Financing the transition
- Bottom-up approach
- Security selection
- Net zero **transition** metrics

$1 - \alpha(t)\%$

$\alpha(t)\%$

Thank you!

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