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An Introduction to Phased Array Design

A Technical Note by N. Tucker

ABSTRACT

There are many texts and publications that deal with the theory of phased array design and some hint at the practicalities. In many cases the topics are advanced and treatment is justifiably complex. However, a beginner can quickly find himself or herself staring into the mathematical abyss and still be left wondering how to actually go about designing an array.

The aim of this note, by way of an example, is to show how theory, modelling and measurement come together in practice. The example used is a 6-element linear array of dipoles over a finite ground-plane, fed using a reactive power splitter. By analysing and solving the problems encountered in this simple example, it is hoped to give the reader a basic framework from which to view more complex analysis.

CONTENTS

1. INTRODUCTION

Although array design is a wide topic in its own right and some designs have hundreds or thousands of elements, even a simple example can provide useful insight into many of the problems encountered.

With the plethora of electromagnetic design packages and theoretical papers that are now available it is easy to get lost in the minutia of a feed point detail, a microstrip mitre or just plain lost in the maths. However, it is aperture size, element spacing and the coupling interactions between all the array elements that governs the fundamental performance of an array. Bringing all these aspects together into a working design is not trivial, but it can be broken down into manageable pieces.

By looking at how each of these ‘pieces’ affects the overall array performance and how they interact with each other we can get a good understanding and ‘feel’ for how an array works. This is essential for robust designs that are tolerant to manufacturing variations. It also it allows physical design, computational and theoretical effort to be concentrated effectively.

In this 6-element design example the fundamental parameters of array pattern, element spacing, phase and amplitude taper are defined using some simple array factor software. Having selected the amplitude and phase excitation for each element, the effects of mutual coupling are examined, together with correction strategies for this coupling. Next a reactive power splitting network is designed. A reactive network was chosen, as opposed to a Wilkinson splitter, because it highlights important issues regarding the effects of mutual coupling and because it is the preferred method for high power use.

A wire grid model of the antenna is developed during the design process using NEC2. This is used to verify the array factor results and evaluate coupling interactions, and finally model the performance of the complete array including feed network. During the process, some of the fundamental problems in array design, and how to solve them, are illustrated.

2. BACKGROUND

Before getting stuck into a specific example, it is probably worth considering how arrays fit in with other types of microwave antenna. The comparisons will help in gaining an understanding for how the array works, as well as its advantages and limitations.

All antennas can be considered to have an ‘aperture’, a term usually qualified by the additional words ‘physical’ or ‘effective’. The physical aperture is just as you would imagine, it is the physical area of the antenna as viewed by a distant observer in the direction of maximum radiation. The effective aperture is the area over which power is extracted from the incident wave and delivered to the load. Derivations and a detailed explanation can be found in [1] and [2].

Although the definition of effective aperture refers to an antenna’s signal collecting ability, the reciprocal nature of antennas means that effective aperture also refers to its transmitting qualities. This is easiest to visualise in the case of classic microwave antennas such as horns, slots and dishes where the effective and physical apertures are more closely related.

If we take the example of a wave-guide horn antenna, the physical aperture is just the area of the open end of the horn. Over this area there exists an electromagnetic field distribution, how this field varies across the aperture determines the radiation characteristics of the antenna. In fact the relationship between this or any aperture field and the corresponding far-field pattern is so close, that some bright spark in the 1930’s realised that they were directly related by the Fourier transform. This almost immediately gave rise to lots other useful deductions regarding antenna apertures and their radiation patterns. These included direct calculation of beam-width, side-lobe levels, null-positions and directivity.

To see how arrays fit in with all of this, and in the interests of not over complicating matters, we can start with the slightly contrived example shown in figure 2-1. This represents a uniform aperture distribution in an infinite ground plane. Using the Fourier transform technique, the equations for the corresponding far-field pattern (Eqs 2-1 and 2-2) can be derived. Note the appearance of the sin(x)/x terms, the Fourier transform of a rectangular pulse. The additional sin(Φ) and cos(θ)cos(Φ) terms at the beginning of the equations are there to resolve the E-field vector into E_θ and E_Φ components in the far field.

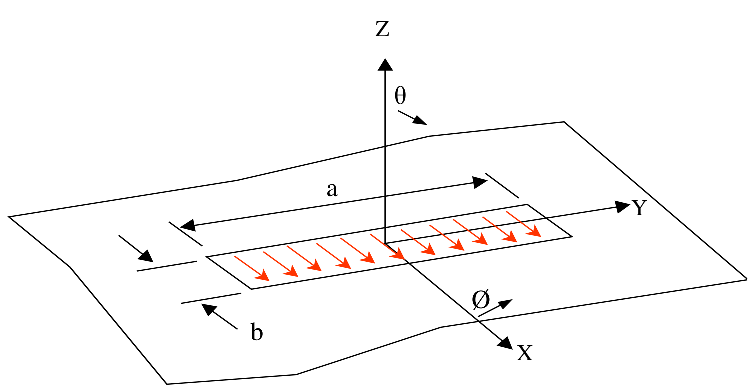

Figure 2-1 Uniform aperture distribution in infinite ground plane

$$E_\theta = \sin(\phi) \cdot \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$ Eq 2-1

$$E_\phi = \cos(\theta) \cdot \cos(\phi) \cdot \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$ Eq 2-2

Where :

$$X = \frac{ka}{2} \cdot \sin(\theta) \cdot \sin(\phi)$$

$$Y = \frac{kb}{2} \cdot \sin(\theta) \cdot \cos(\phi)$$

$$k = \frac{2\pi}{\lambda}$$ *Propagation factor*

Now if we choose the dimension *(a)* of our aperture to be 4λ and dimension *(b)* to be 1λ we can use Eqs 2-1 and 2-2 to calculate the far-field patterns, figure 2-2 (crosses). Now let us take the same aperture and fill it with an array of very short (λ/100) dipole sources, spaced 0.1λ apart. Calculating the far-field patterns from them using ArrayCalc [4], we get figure 2-2 (solid line). It appears we can approximate a continuous aperture-field distribution by an array of individual sources. Depending on your background, this will probably bring to mind discrete Fourier transforms or Huygen's principal.

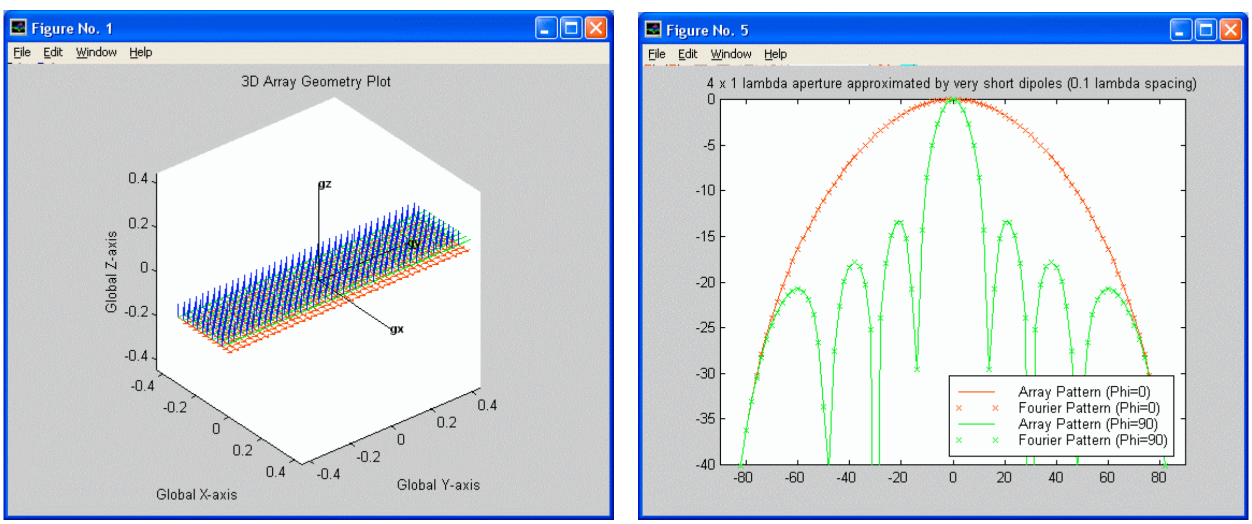

Figure 2-2 Array [10x40] of very short (λ/100) dipoles approximating a 1λ x 4λ aperture.

(ArrayCalc File : ExAp1.m)

As with any approximation, it is of course interesting to understand the limiting factors. To simplify things still further, the dimension *(b)* can be made very small (λ/100) so the array becomes a single 4λ -line of short dipole sources. The graph in figure 2-3 shows the far-field patterns as the number of sources is reduced. The approximation appears to start breaking down at around 10 sources, a spacing of 0.4λ. Once the spacing reaches 0.8λ there is a dramatic departure from the Fourier solution, known as a 'grating lobe'.

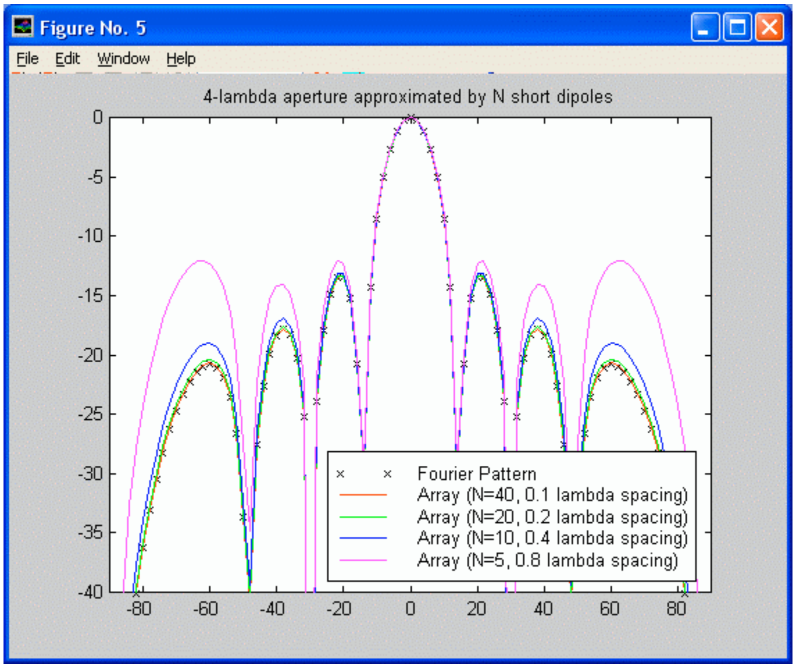

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Figure 2-3 A 4λ x 1/100th λ aperture approximated by a single line of N very short dipoles.

The fact that very few sources (therefore wide spacing) are required for a reasonable approximation is quite handy. This is because most practical radiating elements become quite inefficient once the resonant length drops below 0.5λ, so to avoid overlapping spacing ideally needs to be greater than 0.5λ (see Note**). In practice element spacings of 0.6-0.7λ are common, using amplitude taper to suppress side-lobes close to the main beam can also control the grating lobe to some extent.

Identifying a 'ball-park' value for the element spacing is really the first step in our quest for a practical design. Generally speaking, the fewest number of elements you can get away with and still achieve the required patterns the better. The reasons are two-fold: Firstly, the fewer elements there are the less costly it will be to produce. Secondly, the further apart they are the less coupling there will be between the elements.

Note** This assumes wide-band air spaced elements such as patches or dipoles. For dielectrically loaded elements such as microstrip patches the elements can be resonant with smaller physical dimensions. This allows array spacing of less than 0.5λ, which may well be necessary to prevent grating lobes at large scan angles.

3. DEFINING THE ARRAY

The main points so far are that an array can be considered as an aperture and that the aperture can be approximated by a number of discrete sources. In addition we can say that the larger the aperture dimension, the narrower the beam will be in that axis and visa versa. Also, the larger the aperture is overall, the greater the overall directivity will be. Since we now have an idea of how many elements we will need 'fill' a given aperture we start thinking about our design.

To begin with there are some 'rule of thumb equations', that are useful in deciding roughly what aperture size we will need. (see file Aperture_Calculation.xls)

<u>Equations for Aperture Size Calculation</u>

For a uniformly illuminated aperture L meters long (in x or y dimension) operating at wavelength λ (m).

$\theta_3 = (0.886\ \lambda / L) \cdot 180/\pi$ (θ_3 = 3dB beam-width in degrees, broadside)

$\theta_3 \approx \theta_3(\text{broadside}) / \cos(\theta_0)$ (θ_0 = Scan angle off bore-sight)

$D = 10 \text{ Log}_{10} (32{,}400 \cos(\theta_0) / (\theta_{x3}\ \theta_{y3}))$

Where : (D is directivity in dBi and $\theta_{x3}\ \theta_{y3}$ are the x and y 3dB beam widths in degrees)

For a linear array in x, the y 3dB beam-width will be that of your chosen array element.

Using these equations we can adjust our aperture until we get roughly the beamwidths and directivity we require from our array. Bear in mind however that you will need 'fill' this aperture with a whole number of resonant elements of finite size.

We have already noted that an element spacing of around 0.7λ is about the maximum before we get grating lobe problems. This limits our aperture dimensions to multiples of 0.7λ in both x and y dimensions. Let us assume that an aperture of 4.2λ (6 elements) by 0.7λ (1 element) nominally meets our directivity and beam-width requirements.

From the equations above :

X dimension (0.7λ) 3dB beamwidth 72.5 deg

Y dimension (4.2λ) 3dB beamwidth 12.1 deg

Directivity (dBi) 15.7 dBi

Next, using some simple array-factor software such as ArrayCalc [4] we can trade-off parameters such as side-lobe level and beam-width, directivity and element type/numbers/spacing. For this example we will assume that a vertical array of 6 horizontal λ/2 dipoles will meet our needs. To highlight the design problems we will also assume that some side-lobe reduction is required and that the main beam should electrically (as opposed to mechanically) down tilted by 15deg. Finally, to keep the numbers straightforward we will design at 1GHz.

In terms of specification, this type of antenna is typical of those used in base-stations for mobile phone coverage. The choice of horizontal dipoles is not the norm but to play 'devil's advocate', since this arrangement will have significant broadside coupling between the elements and accentuate the problems.

To summarise :

Array Type	Linear, vertical
Array Element	Half-wave Dipole
Polarisation	Horizontal
Number of Elements	6
Side-lobe Suppression	-20dB 1st side-lobe w.r.t main beam
Down-tilt (Electrical)	15 Deg
Frequency	1GHz
Vertical 3B BW	12.1 Deg
Horizontal 3dB BW	72.5 Deg
Directivity	15.7 dBi

3.1 Initial Modelling

Having decided on a specification we can model a basic configuration. To begin with we will start with an array of 6 horizontal half-wave dipoles, 0.7λ spacing and uniformly excited over a ground plane. In order to try and reduce the element count to a minimum we will try 0.7λ spacing to begin with, keeping an eye out for the 'grating lobe'. The geometry and resulting orthogonal theta pattern cuts (for phi=0 and 90) are shown in figure 3.1-1.

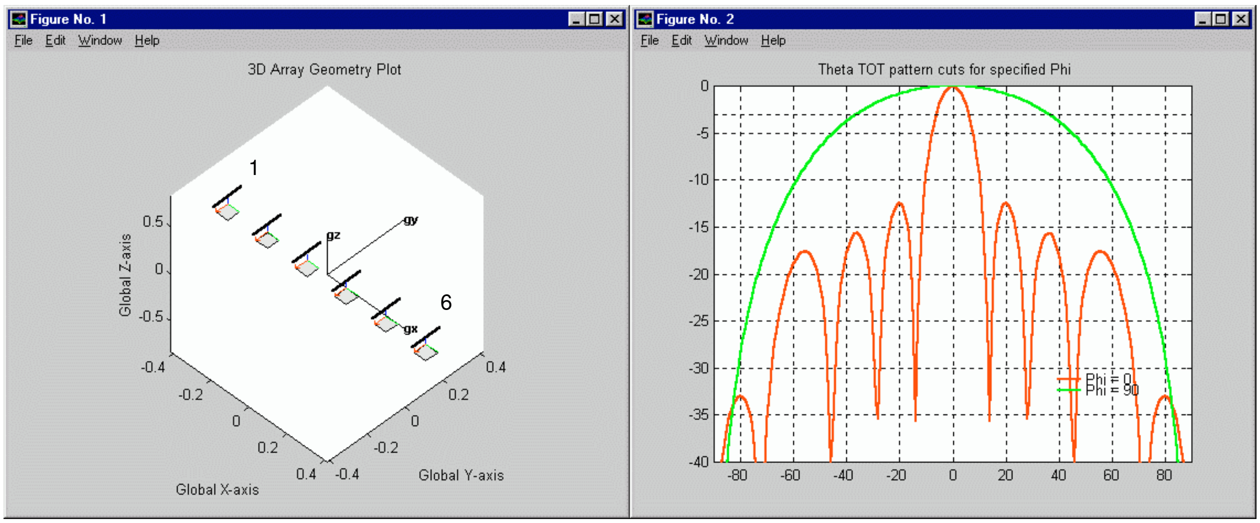

Figure 3.1-1 Basic 6 dipole-over-ground array geometry and radiation patterns.

In the Y-Z plane the aperture is narrow and the pattern is commensurately broad. In the X-Z plane the aperture is 6 x 0.7λ = 4.2λ and has the characteristic sin(x)/x form. The first thing to note is that the side-lobe levels are the higher than we would like. For large apertures with a uniform distribution the sin(x)/x form gives 1st side-lobe levels of 13.2dB below the main beam, this is pretty much what we have.

There is no evidence as yet of our troublesome friend the 'grating lobe'. This is because the dipole has been placed 0.25λ above a ground plane, creating an 'image' of itself 0.25λ below the ground plane and 180˚ out of phase. This effectively produces a 2-element array at each dipole location, which is directional, unlike a single dipole in free-space. See[3] for details of the model. The resulting array pattern is the product of the 2-element pattern and the basic array pattern (produced by omni-directional elements). It is the roll-off of the element pattern that suppresses the array-factor grating lobe.

Array Pattern = Element Pattern x Array Factor

At this stage :

- 3dB beam width is 12.2deg for the phi=0 cut.
- Directivity is 16.0dBi.
- 1st Side-lobe is 12.5dB down on main beam.

So far so good, now to deal with the side-lobe levels. Just as windowing functions are used in Fourier signal analysis to reduce unwanted responses, so they can be used modify aperture distributions to reduce unwanted side-lobes. In theory many distributions are possible however there are two that are frequently used in array design, the Dolph-Tchebycheff and the modified Taylor distributions.

Dolph-Tchebycheff

This distribution is based on the Tchebycheff polynomial and gives a uniform side-lobe reduction ratio for all side-lobes. It also results in the minimum main beam width for a given aperture size and side-lobe level reduction. The uniform side-lobe height makes it a popular choice as a starting point for more complex beam synthesis techniques such as adaptive beam forming.

Since this distribution is based on polynomial coefficients rather than a continuous function, elements are assumed to be discrete and regularly spaced. A procedure to calculate the required coefficients is given in appendix A.

For our application the uniform side-lobe level is not ideal. This is because the electrical beam tilt, mutual coupling and manufacturing tolerances tend to have a greater effect on side-lobes away from the main beam (e.g. 3rd,4th side-lobes). As a result we will be looking to reduce 3rd,4th.etc side-lobes more than the 1st , the modified Taylor distribution is better suited to this requirement.

Modified Taylor Distribution

The modified Taylor distribution is based on a continuous function and therefore can be used to calculate element excitations according to their distance from the centre of the array. The side-lobe reduction ratio refers to the 1st side-lobe, the rest decrease in a similar fashion to those given by a uniform distribution. The equations used to calculate the element excitations are given in appendix A. The graph in figure 3.1-2 shows the effect of applying the distribution to our 6-elemnt array, giving a 1st side-lobe of 20dB. The value actually achieved is 19.5dB, the error is because the distribution is based on a continuous function and the array just samples it at the element locations.

Element Excitations

Ele No.	Power (dB)	Phase (Deg)
1	-5.81	0
2	-1.78	0
3	0.00	0
4	0.00	0
5	-1.78	0
6	-5.81	0

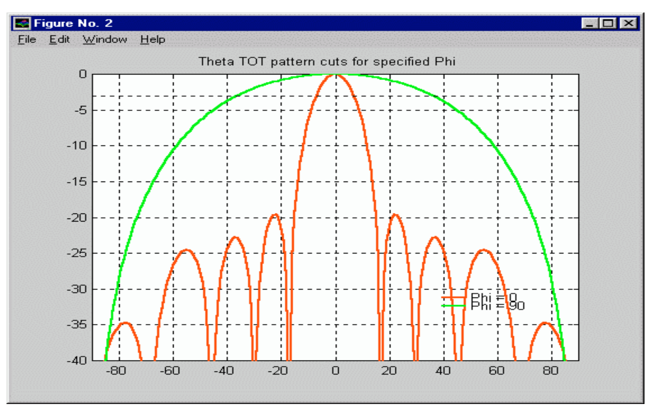

Figure 3.1-2 6-element dipole-over-ground array with modified Taylor amplitude taper.

After applying the amplitude taper the radiation pattern has the following characteristics :

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- 3dB beam width is 14.0deg for the phi=0 cut. (12.2 deg with no taper)
- Directivity is 15.8dBi. (16.0dBi no taper)
- 1st Side-lobe is 19.5dB down on main beam. (12.5dB no taper)

One of the side-effects of using amplitude taper to control side-lobe levels is a broadening of the main beam and loss in directivity of the pattern overall. It comes under the general heading 'you don't get something for nothing'.

<u>Electrical Tilt</u>

The next stage is to apply the 15 degrees of electrical tilt, by altering the phase excitation of the elements. Steering the main beam is simply a matter of adjusting the phases to conform to the relation : *Phase in radians for element n = k.n.d.sin(θ)*

Where k = propagation factor $2\pi/\lambda$ (λ in meters)

n = nth element

d = element spacing (m)

θ = steering angle

The origin of this relation drops straight out of the array geometry and is well documented in most antenna texts.

The graph in figure 3.1-3 shows the 6-element array with amplitude taper and electrical tilt applied (orientation of elements 1-6 left to right on plot). The phases are given modulo(360) so that they are always positive and correspond to adding transmission line in the feed network. (More on this in the section about the feed network).

Element Excitations

Ele No.	Power (dB)	Phase (Deg)
1	-5.81	0.00
2	-1.78	65.32
3	0.00	130.59
4	0.00	195.82
5	-1.78	260.99
6	-5.81	326.11

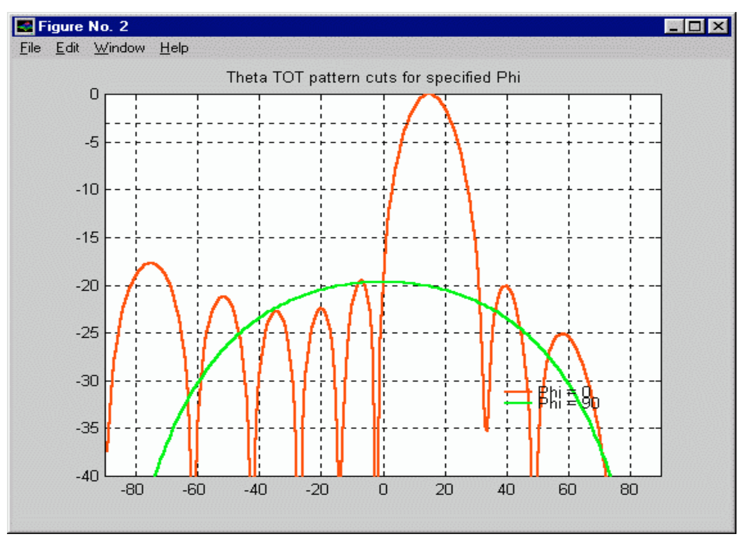


Figure 3.1-3 6-element array with amplitude taper and 15 deg electrical tilt.

Well we definitely got our 15 degrees of tilt, but it appears to have caused a few problems in the process.

First the phi=0 cut level has dropped relative to the main beam, this is because the cut now intersects the side of the main beam rather than its peak. Something to watch out for when making array measurements on a test range, it is easy to forget.

Next, our friend the grating lobe has appear again, on the opposite side of the pattern to the direction of the beam tilt. This is a standard problem that faces array designers and begins to show why so much effort has been put into algorithms to calculate element excitations that reduce these effects. For our purposes it is enough to show the need for such algorithms, since the subject is vast in its own right.

However, at a more basic level we know that element spacing has a dramatic effect on grating lobes, so reducing the element spacing is certainly worth a try. The graph in figure 3.1-4 shows the 6-element array with amplitude taper and element spacing reduced from 0.70λ to 0.65λ. The amplitude taper has also been adjusted slightly by stipulating 20.5dB side-lobe ratio, to bring the 1st side-lobe to exactly 20dB.

Element Excitations

Ele No.	Power (dB)	Phase (Deg)
1	-6.19	0.0
2	-1.88	60.65
3	0.00	121.26
4	0.00	181.82
5	-1.88	242.43
6	-6.19	302.82

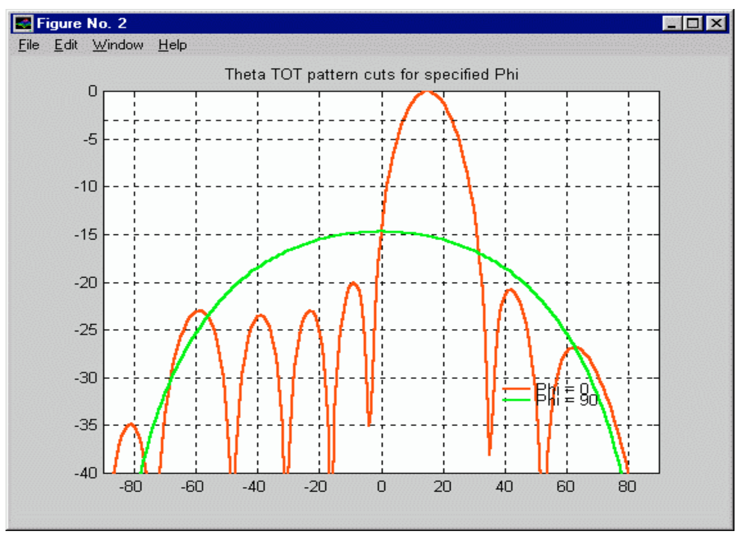


Figure 3.1-4 6-ele array with ampl taper, 15 deg elect tilt, 0.65λ spcing. (File : ExDip1.m)

After the modifications the radiation pattern has the characteristics listed below. Note the further reductions in directivity and increased beam width, due to the reduced aperture size (closer element spacing with still only 6 elements) :

- 3dB beam width is 16.0deg for the phi=0 cut. (14.0deg for 0.7λ spacing version)
- Directivity is 15.3dBi. (15.8dBi for 0.7λ spacing version)
- 1st Side-lobe is 20.0dB down on main beam. (19.5dB for 0.7λ spacing version)

This simple model of the array shows what performance could be expected if the physical implementation was perfect and achieved exactly the right element excitations. In practice of course this will never happen and design margins have to be built in. Despite this, it makes a useful baseline from which to proceed and refer back to if there are problems further down the line. The final array excitations, after this initial modelling are shown in table 3.1-1 below.

In general, for this type of antenna at the centre frequency, I would expect a good implementation to give 1st side-lobes to within +/-2dB (ie. 18-22dB and all others<20dB). A good low loss feed network and element design could reduce ohmic, dielectric and mismatch losses to around 1.0dB giving a potential gain of 15.3dBi –1.0dBi =14.3dBi. For other array configurations (e.g. microstrip patch) losses may be considerably higher and need to be factored into the design. Aspects of good feed network and element design are discussed in later sections.

As a final remark concerning element phase/amplitude excitations, there is often a temptation to try and be too clever. Improved performance may be possible using automated optimisation of element excitations, but this can be risky. The result can be unstable, small errors due to manufacturing tolerance can result in significant errors in the patterns produced. This is due in part to the other problem, which is that certain amplitude distributions are physically difficult to realise. For example highly unequal power splits in the feed network or high power elements at the edge of arrays.

The slightly 'plodding' design process described for this example may not give the maximum performance possible from the available aperture, but it is fairly robust, and the sources of problems are likely to be easier to spot.

<u>Array Excitation Values</u>

Element No.	Ampl(dB)	Phase(Deg)	Real(Lin)	Imag(Lin)
1	-6.19	0.00	+0.490	+0.000
2	-1.88	60.65	+0.395	-0.702
3	0	121.26	-0.519	-0.855
4	0	181.82	-0.999	+0.032
5	-1.88	242.34	-0.374	+0.713
6	-6.19	302.82	+0.266	+0.412

Table 3.1-1 Array Excitations

Note : Phase(Deg) column represents phase delay, hence the Re,Im columns are calculated using Amp(Lin)*(cos(-Phase)+j*sin(-Phase))

4. DEFINING THE ARRAY ELEMENT

In our example we have already defined the array element as being a dipole over a ground plane. In practice the choice of array element and technology used to fabricate the array is governed by many factors, some of which are listed below. In fact depending on the design, these parameters are just as valid for designing the feed network used to excite the elements.

<u>Design Factors</u>

- Frequency
- Bandwidth
- Polarisation
- Scanning requirements
- Environmental
- Mass
- Physical profile
- Cost

To choose an element, comparing available designs and manufacturing methods against the list above is not a bad place to start. Obviously it is not possible to cover all the permutations and combinations here, but assuming you have decided on the element type, it is important to understand how it behaves. Of particular relevance is how to tune the element to the desired frequency and input impedance, and the sensitivity of these parameters to the dimension changes.

While this may seem rather obvious, bear in mind that you may not have the luxury of being able to measure its impedance or resonant frequency directly. It is therefore important that you are able to make tuning adjustments ‘open loop’ or with inferred data from other measurements. For example you may find that pattern data from your computer model / prototype of the full array indicates that it is operating 2% low in frequency. If you are then going to the effort of re-tuning all the elements or building a new model / prototype you want to be pretty sure you have made the right adjustments.

4.1 The dipole element

The obvious starting point for a thin-wire, half-wave dipole design is to set the length to λ/2 (at 1GHz in free space this equates to 150mm). This λ/2 design will be referred to as Dipole-100%, modifications to the length will be defined as percentages of this nominal value.

<u>Dipole-100%</u>

Using the NEC2 code we can model this thin dipole in free space and look at its input impedance, labelled ‘Dipole-100 pcnt’ in figure 4.1-1. Notice that the dipole is resonating rather low in frequency. This is due to the fringing fields at the ends of the dipole, making it longer than it actually is.

Dipole-95%

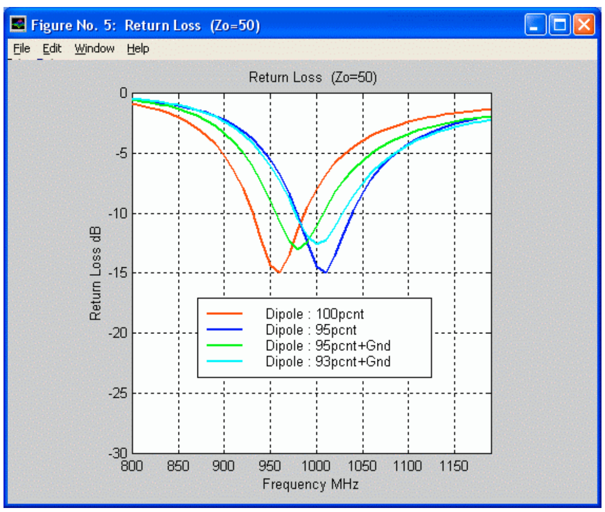
Reducing the dipole to 95% of the λ/2 value compensates for the fringing fields and brings the resonance back to 1GHz. Note there is little effect on the real value of the impedance.

Dipole-95% +Gnd

If we now place a ground plane λ/4 below the dipole, the dipole has been de-tuned again to a lower frequency. Notice also that the real impedance value has been changed to around 80ohms, compared to the theoretical free space value of 73ohms.

Dipole-93% +Gnd

Again this can be corrected by shortening the dipole, now to 93% of its λ/2 value. Notice that the impedance value has remained at around 80ohms. Of course in practice this will have to be taken into account or corrected by modifications to the dipole's balun/transformer.



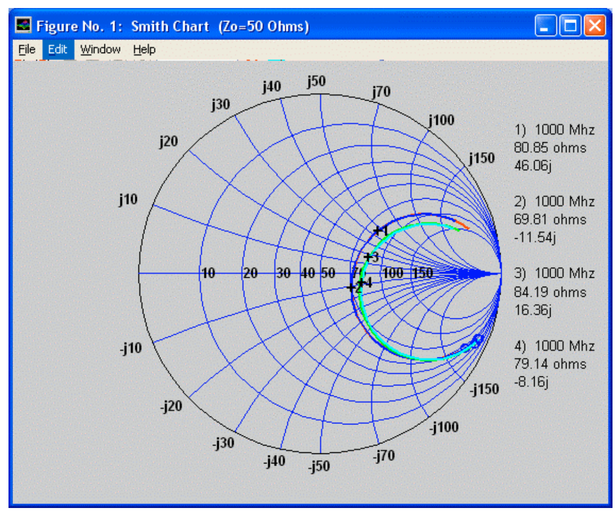


Figure 4.1-1 Dipole Return Loss and Smith Chart

The 93% of λ/2 length for the dipole corresponds to 139.5mm at 1 GHz. Even at this early stage in the design we can see that the immediate environment around the element has had a significant effect on its operation. If your antenna is destined to have a radome to protect it, this is the time to include it in the model or make some measurements of a prototype.

While a radome can be designed to be almost transparent to RF at certain angles of incidence, this assumes plane wave incidence (i.e. in the far field). Many radomes actually lie in the reactive near-field of the antenna and thus form an integral part of the element 'tuning'. I can't emphasise enough how important it is to try and include the radome effects at an early stage. I have seen many designs fail, either because someone has 'plonked' a radome over the antenna as an after-thought or just arbitrarily changed its shape or height above the array.

5. MODELLING THE ARRAY USING NEC2

Having defined a nominal array and element configuration, the next logical step is to put the two together. NEC2 is a convenient tool for exploring the array performance, including the effects of mutual coupling, the wire grid model is shown in figue 5-1 below. Note that the orientation has been changed so the array is now vertical in the global x,y,z axis system. This is so the (Ephi & Etheta) pattern data that NEC2 outputs will actually represent Horizontal (Co-polar) and Vertical (Cx-polar) field components respectively. Boresight for the array with tilt will now be at (theta=15+90=105 , phi=0).

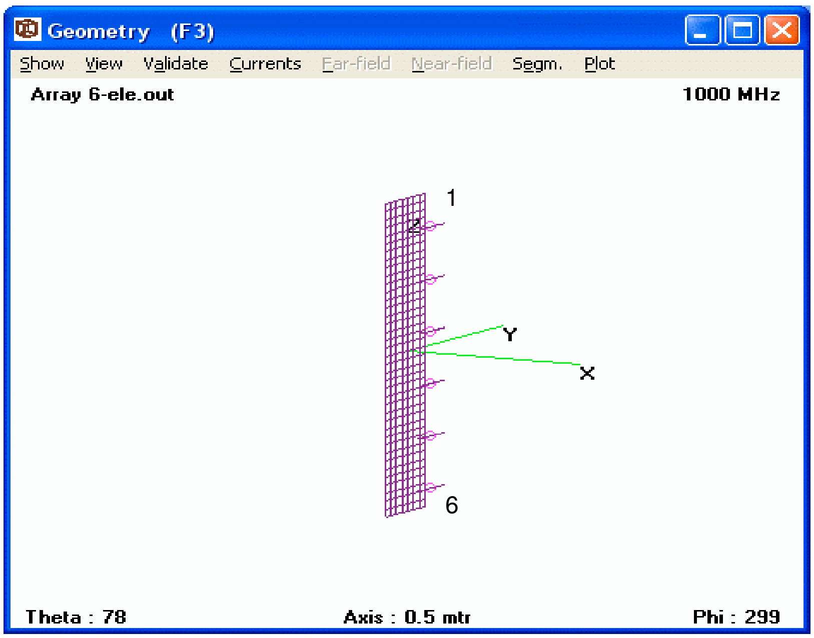


Figure 5-1 NEC2 Model

However, before diving in too deeply it may be wise to start with a slightly simplified example. In this first run of the model there is no electrical tilt (phase taper), just amplitude taper.

5.1 Amplitude taper

In this example the following configuration was modelled :

- Frequency 1GHz (Pattern) and 0.5 to 1.5Ghz (Impedance)
- 6 Dipole elements (21 segments each, Excitation on segment 11)
- Spacing : 0.65 Lambda
- Amplitude Taper : as per Table 3.1-1 dB column. (0.49 0.805 1 1 0.805 0.49) Re Lin
- Phase taper : None, hence the Real Linear component values above are different to the Real Linear values listed in table 3.1-1
- Elements set 0.25 Lambda above finite gridded ground-plane of dimensions 0.7λ by 3.9λ (6x0.65), (ref Figure3.1-1 for element orientation and numbering)

The graphs in figure 5.1-1 show the array pattern and the element impedances for each element.

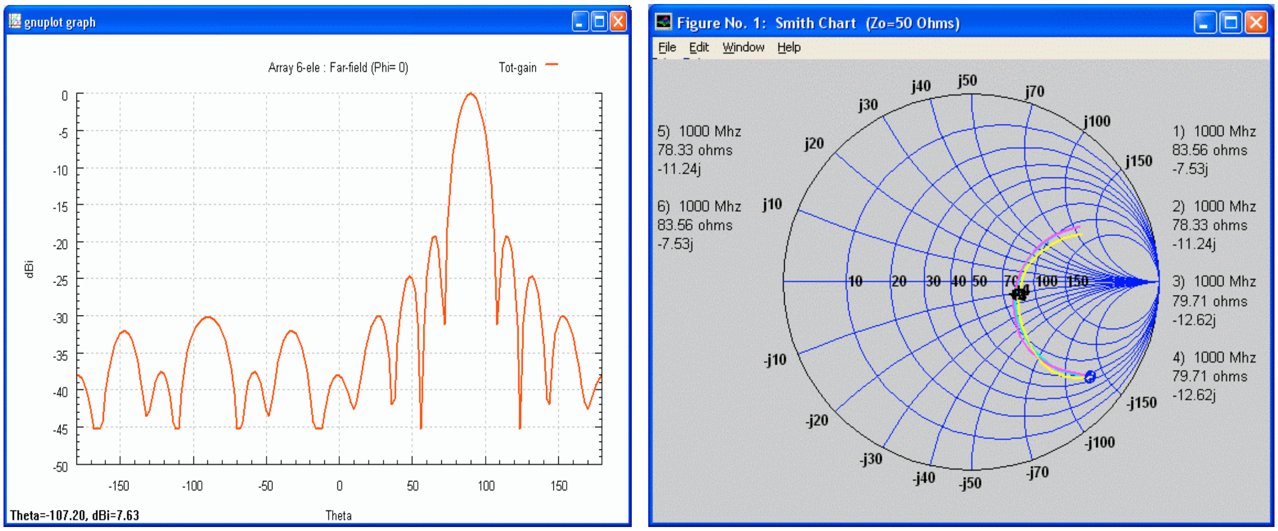


Figure 5.1-1 Array pattern and element impedances for file : Array 6-ele a.nec

So far so good, the pattern looks OK (sidelobe levels are good) and the element impedances are all similar, between 78 -11j and 84 –7j Ohms.

5.2 Amplitude and Phase Taper

Now to make things more interesting let us add the electrical tilt (phase taper). The array excitation is now exactly as per table 3.1-1. Figure 5.2-1 below shows the array pattern and element impedances.

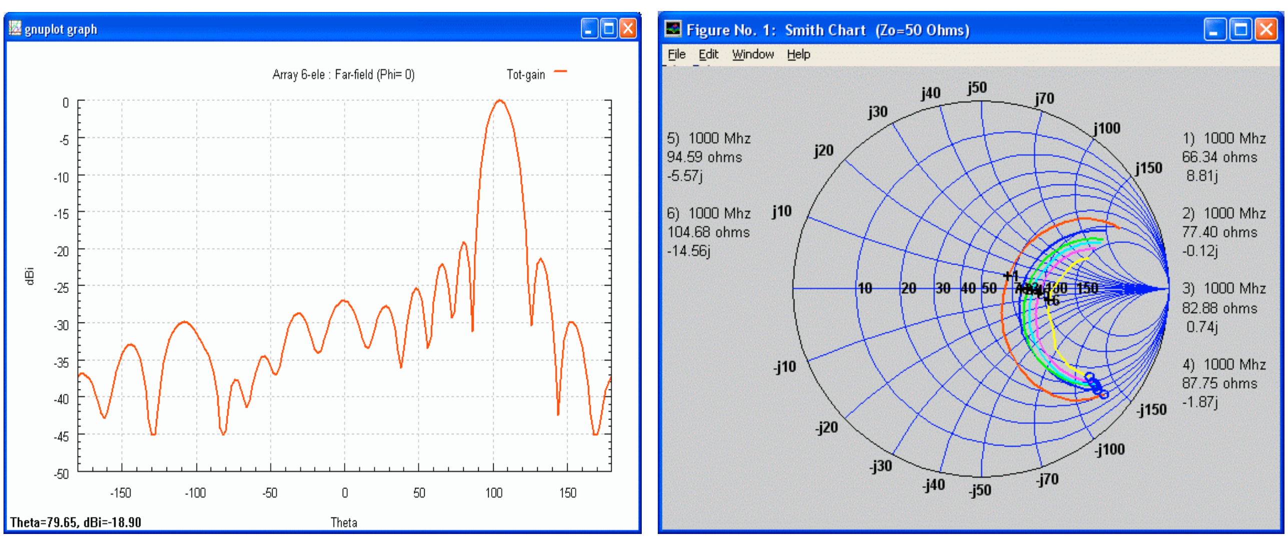


Figure 5.2-1 Array pattern and element impedances for file : Array 6-ele b.nec

As you can see, this is where the fun starts!

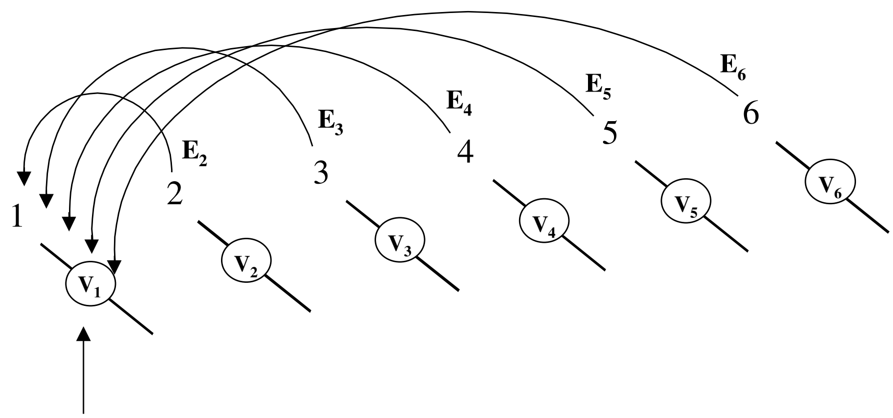
In the previous example the symmetrical nature of the excitations resulted in the mutual coupling balancing out, and those effects that did exist were symmetrical across the array. Now a progressive phase taper has been introduced, the full horror of what awaits the would-be array designer becomes apparent. Just looking at the element impedances, which now range from 66+9j to 105-15j Ohms, it is not difficult to imagine the potential problems that would be caused in a reactive power splitting network.

The pattern still looks quite good because in this model because the array excitations are ‘forced’ by each element having its own source, with defined amplitude and phase. In our array, the power applied to the elements will ultimately depend upon the impedances presented at each junction of the power splitter. If the splitter was designed assuming that all element impedances were the same then there would be a surprise in store.

Having identified such a fundamental problem we really need look a little more closely at what causes these changes in element impedance. The next section deals with ‘active input impedance’ and how it is calculated.

6. ACTIVE INPUT IMPEDANCE

As we have seen in the previous section, coupling between the elements in the array has a significant effect on the element's impedance. To understand what is happening a diagram will be helpful, figure 6-1 shows the array configuration, the various element excitation signals and the coupling paths.



$$Z = V / I$$

$$Z_1 = (V_1 + E_2 + E_3 + E_4 + E_5 + E_6) \ / \ I_1$$

Figure 6-1 Array mutual coupling paths for element-1

In our NEC2 model of the array each dipole was divided into 21 segments and a voltage source assigned to the centre segment (segment 11) of each dipole. This allowed a specific amplitude and phase excitation to be applied to the individual elements.

When NEC2 is run a large set of simultaneous equations are produced, defining the field interactions between each segment and every other. These equations are then solved to find the current induced on each of the wire segments in the model. Since we know the voltage and the current on the centre (driven) segment of the dipole the input impedance can be calculated.

However, as you can see from Figure 6-1 the voltage source V_1 is not the only electric field acting on dipole-1. The radiated fields from all the other dipoles in the array are also inducing currents in dipole-1 and hence affecting its impedance. Because the E-fields (E_1, E_2… E_6) add as vectors, the total E-field and therefore current induced in dipole-1 depends on not only the phase and amplitude of the other voltage sources but also the physical location of the other dipoles.

The problem is that although we have been able to calculate the impedance for each array element, the solution is only valid for one set of phase and amplitude excitations. If we change any of the excitations we will have to re-calculate the entire problem again, even if the location of the dipoles is unchanged. Since the object of many phased array designs is to electronically scan the array by changing the amplitude and phase excitations, it would be really useful if the effect of excitations could be separated from the array geometry. In this way we would only have to do a potentially time consuming NEC2 type calculation once for a given array geometry.

One way of separating the effects is to treat the array as multi-port network and characterise the interaction between the array elements using S-parameters.

For a standard 2-port network :

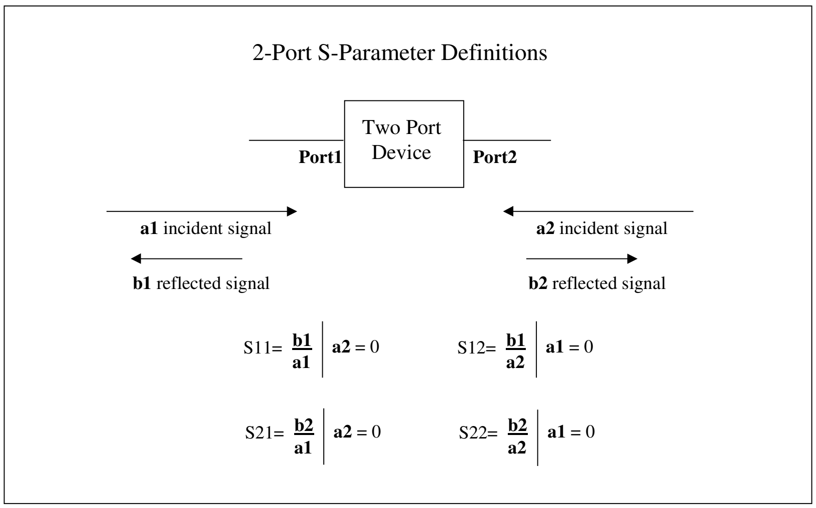


Figure 6-2 Standard 2-port S-parameters

In the strict definition of the S-parameters the port-1 input reflection coefficient S11 tells us that if a signal a1 is incident on port-1 then b1=S11*a1 will be returned. It is assumed that port-2 is terminated in the characteristic impedance and there is no incident signal on port-2, therefore a2=0.

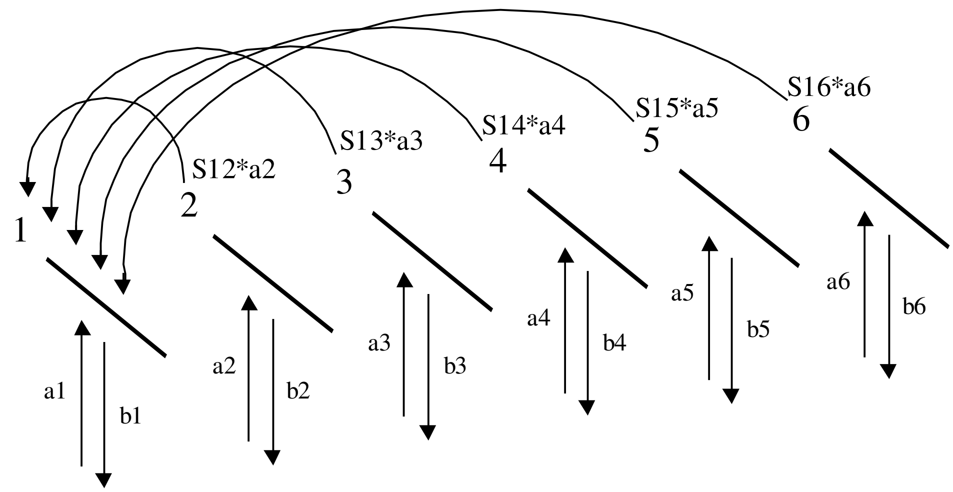
By using the formula below we can calculate the input impedance at port1.

$$Zin = \frac{1+S11}{1-S11} \cdot Zo \qquad \text{Eq 6-1}$$

Now if we take this general idea and apply it to our array geometry we have an arrangement as shown in figure 6-3. Here we have defined a variation of the standard S11 parameter, termed ‘S11active’. In this case however the incident waves on the other ports are not zero, they are the array excitations. The definition for the S11active parameter can be arrived at intuitively by examining figure 6-3.

Giving :

$$S11active = S11 + (S12 \cdot a2 + S13 \cdot a3 + S14 \cdot a4 + S15 \cdot a5 + S16 \cdot a6) / a1 \qquad \text{Eq 6-2}$$



$S11_{active} = b1_{active}/a1$

Where : $b1_{active} = S11*a1 + (S12*a2 + S13*a3 + S14*a4 + S15*a5 + S16*a6)$

Then : $S11_{active} = S11 + (S12*a2 + S13*a3 + S14*a4 + S15*a5 + S16*a6) / a1$

Figure 6-3 Active input impedance for element-1

A more practical form for Eq 6-2 replaces the forward and reflected waves a_n and b_n with amplitude and phase representaions of the signals $A_n e^{j\theta n}$.

$$S11active = S11 + \left(S12 \cdot A_2 e^{j\theta_2} + ...S16 \cdot A_6 e^{j\theta_6}\right) / A_1 e^{j\theta_1} \qquad \text{Eq 6-3}$$

And in a more generalised form

$$Snn(active) = Snn + \left(\left[\sum_{m=1}^{N} Snm \cdot A_m e^{j\theta_m} \right] - Snn \cdot A_n{}^{j\theta_n} \right) / A_n e^{j\theta_n} \qquad \text{Eq 6-4}$$

Note: The -Snn $A_n e^{j\theta n}$ term is needed to remove the extra term introduced by using a generalised summation.

Simplifying Eq 6-4 by performing the division by $A_n e^{j\theta n}$ yields

$$Snn(active) = \left[\sum_{m=1}^{N} Snm \cdot A_m e^{j\theta_m} \right] / A_n e^{j\theta_n} \qquad \text{Eq 6-5}$$

Where : N is the number of elements

A_n is the signal amplitude in linear volts for the n^{th} element.

θ_n is the signal phase in radians for the n^{th} element.

To find the active input impedance for a given array element Eq 6-5 can be used in conjunction with Eq 6-1 to give

$$Znn(active) = \frac{1 + Snn(active)}{1 - Snn(active)} \cdot Zo \qquad \text{Eq 6-6}$$

The importance of active input impedance should not be underestimated. Its effects can be dramatic because it directly impacts the amplitude and phase excitations of the array elements and these are the very parameters you are using to control the array!

We have looked at two methods of finding the active input impedance:

1) By directly by applying the required amplitudes and phases to the elements and examining the resulting input impedances. This requires the whole array to be ‘solved’ for each amplitude and phase configuration.

2) An indirect method using S-parameters, only requiring the array interactions to be calculated once.

The majority of Electromagnetic solvers will readily provide the data for method 1, most will also output a full set of S-parameters if required. My assumption of course is that the reader has no burning desire to write an EM solver from scratch and is happy to use commercial or public domain offerings such as NEC2, Sonnet, HFSS etc

6.1 Using active input impedance

In section 5 we made our first foray into modelling the array to include mutual coupling and identified a fairly significant problem in the form of active input impedance. In section 6 we have found out how to calculate values for the active impedance. Now we must decide how to use the values in our design, to ensure our array functions correctly.

Basically there are two main options :

1) Knowing the active impedance of each element we must match to, we can include a matching section at the input to each element. As long as each element plus matching section results in an impedance locii clustered around the chosen characteristic impedance (e.g. Zo=50ohms); the power splitter can be designed like any other and tested as normal in a 50ohm system. The elements and splitter can then be simply ‘plugged’ together.

   This approach is fine for an array such as ours with fixed element excitations and is well suited to the direct method of active impedance calculation. For an electronically steerable array a different approach is needed, see below.

2) The indirect method of active impedance calculation, using S-parameters, gives us impedance information for all intended array excitations. However, a similar derivation can give an even more flexible method of actually achieving the intended excitation values. Basically it involves using S-parameters to calculate a set of source excitations that result in the correct current values on the array elements, when mutual coupling is taken into account.

Although in the 6-element example we will be looking primarily at option 1. It is worth exploring option-2 a little further, before moving on.

Suppose we were in the situation where we could drive the array elements individually, using a transmit/receive module for example. It is clear that despite applying a particular amplitude and phase to an element, mutual coupling effects mean that we don’t actually achieve the desired excitation. Moreover the excitation we do achieve on one element will depend on how the array is driven overall i.e. The excitation of all the other elements.

What we need to do is calculate a set of 'applied' excitations that will give the correct 'actual' excitations with all the mutual couplings taken into account. Revisiting figure 6.3 gives us a way of formulating a solution in a similar fashion to the active input impedance.

Starting with standard S11 as a ratio of the 'active' forward and reflected waves

$$S11 = \frac{b1_{active}}{a1_{active}} \quad \text{Or} \quad \text{re-arranged} \quad a1_{active} = \frac{b1_{active}}{S11} \qquad \text{Eq 6.1-1}$$

Where

$$b1_{active} = S11 \cdot a1 + S12 \cdot a2 + ...S16 \cdot a6$$

And

$a1_{active}$ Is the wave leaving element 1 in the presence of mutual coupling. This could equally be thought of as the 'actual' excitation achieved and as such will be written as $a1_{actual}$ from now on.

Equation 6.1-1 can thus be expanded to give

$$a1_{actual} = \frac{S11 \cdot a1 + S12 \cdot a2 + ...S16 \cdot a6}{S11} \qquad \text{Eq 6.1-2}$$

A more practical form for Eq 6.1-2 replaces the forward and reflected waves a_n and b_n with amplitude and phase representations of the signals $A_n e^{j\theta n}$.

$$a1_{actual} = \frac{S11 \cdot A_1 e^{j\theta_1} + S12 \cdot A_2 e^{j\theta_2} + ...S16 \cdot A_6 e^{j\theta_6}}{S11} \qquad \text{Eq 6.1-3}$$

Perform the division through by S11

$$a1_{actual} = A_1 e^{j\theta_1} + (S12 \cdot A_2 e^{j\theta_2} + ...S16 \cdot A_6 e^{j\theta_6}) / S11 \qquad \text{Eq 6.1-4}$$

And in a more generalised form

$$A_n e^{j\theta_n}{}_{actual} = A_n e^{j\theta_n} + \left(\left[\sum_{m=1}^{N} A_m e^{j\theta_m} \cdot Snm \right] - A_n e^{j\theta_n} \cdot Snn \right) / Snn \qquad \text{Eq 6.1-5}$$

Note: The $-A_n e^{j\theta n}\, Snn$ term is needed to remove the extra term introduced by using a generalised summation, just as in the active impedance derivation.

Simplifying Eq 6.1-5 and dividing through by *Snn* yields

$$A_n e^{j\theta_n}{}_{actual} = \left[\sum_{m=1}^{N} A_m e^{j\theta_m} \cdot Snm \right] \qquad \text{Eq 6.1-6}$$

Where : N is the number of elements

A_n is the signal amplitude in linear volts for the n[th] element.

θ_n is the signal phase in radians for the n[th] element.

The $A_m e^{j\theta m}$ term inside the summation represents the 'applied' excitations and will be written as $A_m e^{j\theta m}{}_{applied}$ from now on.

Eq 6.1-6 can be rewritten in matrix form as

[A] * [B] = [C]

$[Snm] * [A_m\, e^{j\theta m}{}_{applied}] = [A_n e^{j\theta n}{}_{actual}]$ Eq 6.1-7

Or in explicit form for a 3-element array would look like this :

[A]	*	[B]	=	[C]
S11 S12 S13		$A_1 e^{j\theta 1}{}_{applied}$		$A_1 e^{j\theta 1}{}_{actual}$
S21 S22 S23	*	$A_2 e^{j\theta 2}{}_{applied}$	=	$A_2 e^{j\theta 2}{}_{actual}$
S31 S32 S33		$A_3 e^{j\theta 3}{}_{applied}$		$A_3 e^{j\theta 3}{}_{actual}$

The equation Eq 6.1-7 can be solved by inverting the coupling matrix [A], as in Eq 6.1-8.

We can now calculate the necessary 'applied' excitations [B] that will give the 'actual' excitations [C] . The matrix [C] typically contains design values as defined using simple array factor or pattern synthesis software. E.g. results in table 3.1-1

$[B] = [A^{-1}] * [C]$ Eq 6.1-8

Ultimately the two options described here, for correcting mutual impedance effects, are two sides of the same coin. The first option lends itself to fixed beam-forming networks and is probably a little more intuitive in that it deals with impedances. The second option is better suited to electrically steerable arrays and possibly requires more 'faith' in the maths.

6.2 The 6-element array

Getting back to the 6-element array design, we need to correct for the 'active' impedances we calculated using NEC2 in section 5.2.

In Figure 5.2-1, there is a smith chart showing the active element impedances for our array, as calculated using NEC2. The active impedances are tabulated in table 6.2-1 for each element at the design frequency of 1000MHz (The markers on the smith chart).

In this case, to match the active impedances to 50 ohms (single ended) we need a balun together with an impedance transformer. Ultimately baluns can take many forms, often including an impedance transformation as well as the single ended to differential conversion. I feel that to include balun design detail at this stage would over complicate the problem, so it will be dealt with another time. For now we will assume that the balun is a 'black box' and we will just use a ¼ wave impedance transformer, calculated using Eq 6.2-1. This simple match is useful because it is generic, applying to single ended or differential transmission lines.

$$Z_{match} = \sqrt{\mathrm{Re}(Z_{active}) \cdot Zo} \qquad \text{Eq 6.2-1}$$

The ¼ wave matching transformer given by Eq6.2-1 uses the real part of Z_{active} and therefore will only correct for the real part of the active impedance, the imaginary parts of Z_{active} , if left will cause some phase error. The greatest error is element 6 at –15.6j, however this is also one of the lowest amplitudes at –6.19dB, so will not cause too great a problem.

It should be noted however, if the active input impedances have significant imaginary components then the elements themselves may need re-tuning, reference section 4.1. On balance, looking at table 6.2-1 (or fig 5.2-1) it looks as though increasing all dipole lengths slightly would help even out the +ve / -ve spread of imaginary components. We will leave it for now but keep it in mind for a final tuning option. From a manufacturing point of view it is obviously better to minimise the number of different element configurations, particularly if the elements are fabricated separately.

Element	Real(Ohms)	Imag(Ohms)	Z_{match}(Ohms)
1	66.4	+8.8j	57.6
2	77.4	-0.1j	62.2
3	82.9	+0.7j	64.4
4	87.8	-1.9j	66.2
5	94.6	-5.6j	68.8
6	104.7	-15.6j	72.3

Table 6.2-1 Active input impedances and λ/4 matching transformer values

Prepared By : Neill Tucker	No. 09:003		
Project Title : Array Design	Date 15/04/2011	Rev F	File C:\ My Documents \ Array Design\ Array Design Introduction.doc

If everything has gone according to plan we should now be in a situation where the dipole array will look like an n-port network (Zo=50 Ohms), when the array is excited as per table 3.1-1. The network is illustrated in figure 6.2-1 below.

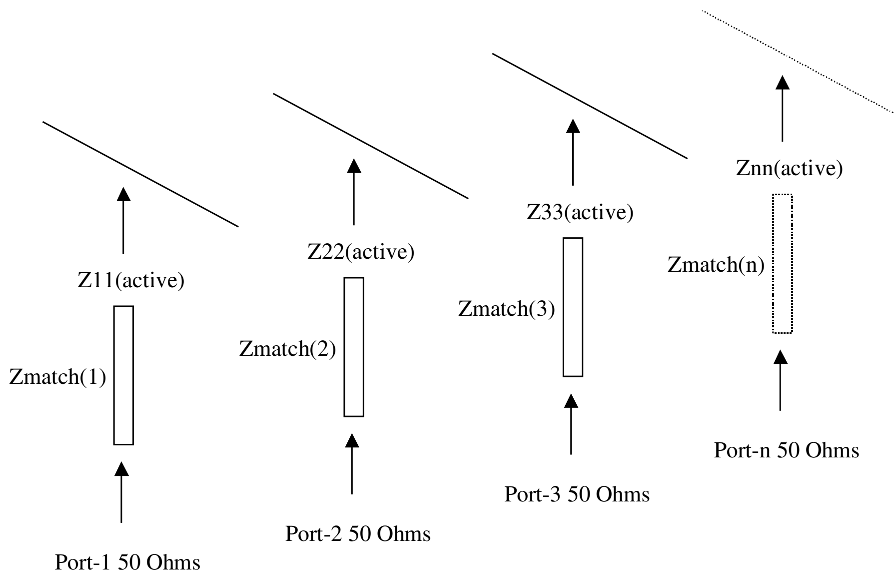


Figure 6.2-1 Schematic showing active impedance correction

It should now be possible to design the power splitter to work in a standard 50 Ohm system and 'plug' it into the dipole array. Fig 6.2-2 below, shows the effect of the impedance matching transformers. The impedances now presented to the power splitter are shown as dotted lines, compared to solid lines before. (also ref back to fig 5.2-1).

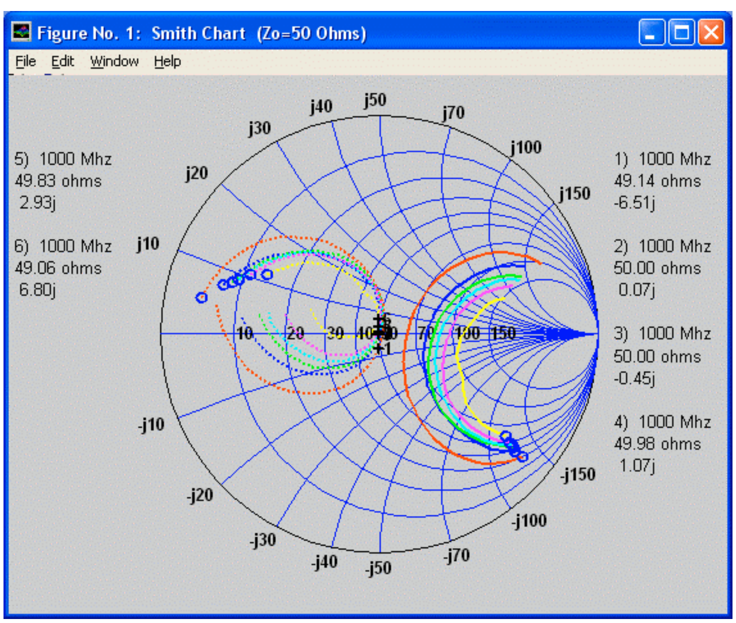


Figure 6.2-2 Effect of impedance matching transformers

7. POWER SPLITTER DESIGN

The function of the power splitter is to take power from a single rf input port and distribute it among output ports in accordance with the desired amplitude and phase distribution.

A typical configuration for a 2-way splitter is shown in figure 7-1. The characteristic impedance Zo arriving at the arms of the splitter is transformed to the junction impedances Z_1 & Z_2 by the ¼ wave line sections Z_1(m) & Z_2(m) respectively. It can be shown that the ratio of the power split is inversely proportional to the impedances (Z_1 & Z_2) and that the impedance at the junction Z_c is simply (Z_1 & Z_2) in parallel.

Ideally Z_1(m) & Z_2(m) are chosen so that Z_1 & Z_2 give the correct power split and combine to give Z_c equal to Zo directly. If it is not practical, then Z_c can be transformed to Zo using a 3rd transformer Z_c(m).

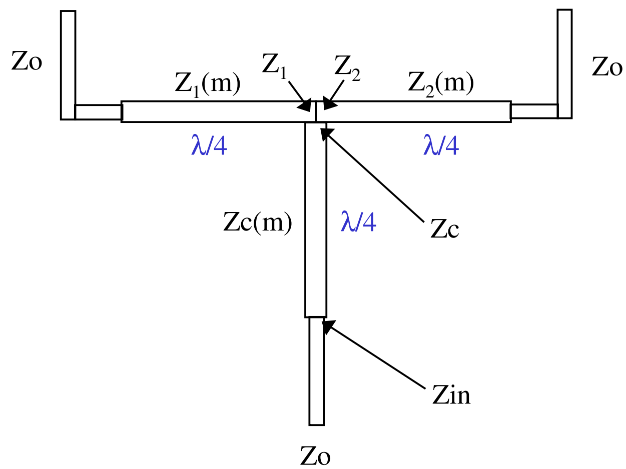


Figure 7-1 2-way splitter configuration

To calculate the required impedances we can use the following :

$$Ratio = \frac{\frac{1}{Z_2}}{\frac{1}{Z_1}} = \frac{Z_1}{Z_2}$$

The linear power split ratio Eq 7-1

$$\frac{1}{Zc} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

The impedances combine in parallel Eq 7-2

From Eq 7-1

$$Z_1 = Ratio \cdot Z_2$$

Eq 7-3

Substituting Eq 7-3 into Eq 7-2 and simplifying

$$\frac{1}{Zc} = \frac{(1+Ratio)}{Ratio \cdot Z_2}$$ Eq 7-4

Re-arranging Eq 7-4

$$Z_2 = \frac{Zc(1+Ratio)}{Ratio}$$ Eq 7-5

For the ¼ wave matching transformers we can use

$$Z_{match} = \sqrt{Z_{req} \cdot Z_{load}}$$ Eq 7-6

Where

Z_{load} is the impedance to be transformed

Z_{req} is the required impedance

Z_{match} is the matching impedance

If we start with the power distribution that is listed in table 3.1-1, entered in table 7-1 below as ‘Pwr(dB) .

Then we convert to linear power Pwr(Lin). Using 10^(dB/10)

Next, normalise the linear power ratios so that the sum of the ‘Pwr(lin) row is equal to 1, giving Npwr(Lin). Normalised by using Npwr(Lin) = Pwr(Lin)/sum(Pwr(Lin)).

Element	**1**	**2**	**3**	**4**	**5**	**6**
Pwr (dB)	–6.19	–1.88	0.0	0.0	–1.88	–6.19
Pwr (Lin)	0.2404	0.6486	1.0	1.0	0.6486	0.2404
Npwr(Lin)	0.0636	0.1717	0.2647	0.2647	0.1717	0.0636

Table 7-1 Power split ratios

This final row in table 7-1 tells us how our unit of rf power is to be split up, and we are faced with some choices. We could design a 6-way splitter to bring all 6 inputs together at once. The problem with this approach will be the impedances we need to present at the power split junction.

In order to achieve high split ratios (such as that between elements 1 and 3) we need high impedance ratios. Also, by bringing all 6 impedances together at one junction, the parallel junction impedance will inevitably be very low. Particularly high or low impedances are best avoided for the following reasons :

High-impedance lines will be thin and therefore the actual impedance values will be difficult to control due to tolerances. The high impedance will also result in high voltages on the line, causing excessive dielectric losses and potential breakdown in high power systems.

Low impedance lines will be wide and potentially take up a lot of board area. They can also present capacitive mismatch problems when forming junctions with narrower lines. The low impedance also results in high current densities and can result in excessive conductor losses.

Exactly what range of impedances is acceptable depends mainly on the transmission line type, production technology used to fabricate it and the required power handling. As a very rough guide try and keep actual line impedances (matching transformers) between 35 and 100 Ohms and impedances presented at the junctions between 25 and 200 Ohms

So, to proceed with our 6-element example we need to decide on a basic topology for the splitter. Based on the comments above it would seem logical to reduce the problem to a series of 2-way power splits. In this way there is the option to transform the junction impedance Z_c back to or near Zo at each split.

The split ratios can be kept to a minimum by using the topology shown in figure 6-2 below.

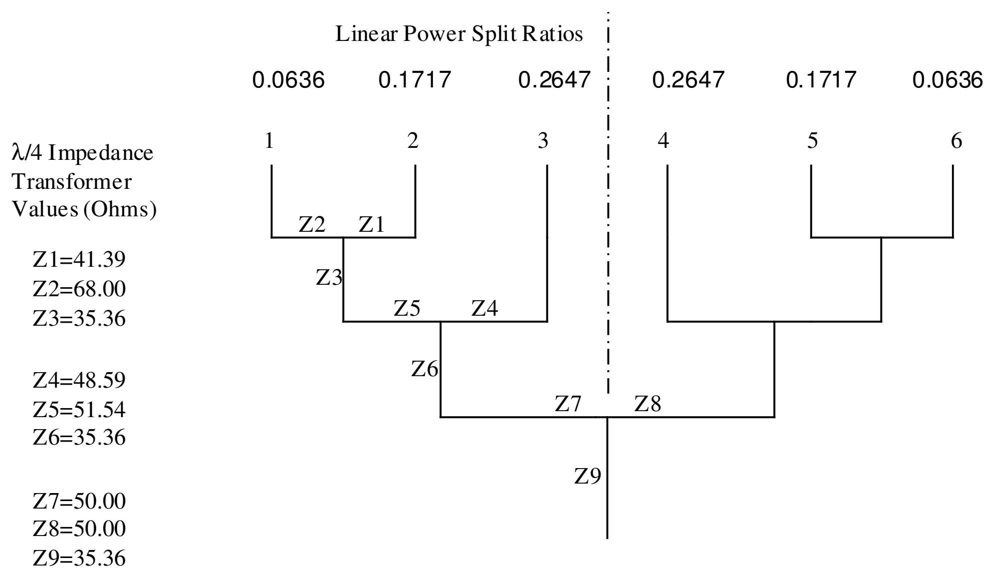


Figure 7-2 6-Way Power Splitter Topology

The impedance transformer values were calculated using Eqs 7-3, 7-5 and 7-6.

For example, the power split for element 1 & 2 was calculated by first normalising the power split ratio with respect to the higher power element (element 2)

Element	1	2
Ratio(Lin)	0.0636	0.1717
Normalised w.r.t 2	**0.3704**	1.0000

To keep the impedance transformer values within reasonable limits the target combined impedance Z_c is set to 25 Ohms.

Using the normalised split ratio (0.3704) and Z_c (25) in equation 6-5 we get

$$Z_2 = \frac{25(1+0.3704)}{0.3704} = 92.5 Ohms$$

This is the junction impedance for LOW power arm of the split.

Now substitute Z_2 into Eq 6-3

$$Z_1 = 0.3704 \times 92.5 = 34.3 Ohms$$

This is the junction impedance for HIGH power arm of the split.

Finally use Z_1 , Z_2 and Z_c in Eq 6-6 to get the impedance transformer values

$Z_2(m) = \sqrt{92.5 \times 50} = 68.0 Ohms$ LOW power branch transformer

$Z_1(m) = \sqrt{34.3 \times 50} = 41.4 Ohms$ HIGH power branch transformer

$Z_c(m) = \sqrt{25 \times 50} = 35.4 Ohms$ INPUT branch transformer

To calculate the next split between elements 1 and 2 combined and element 3 we proceed with :

Element	1 & 2	3
Ratio(Lin)	0.0636+0.1717	0.2647
Normalised w.r.t 3	**0.8889**	1.0000

An so on, as before…..

(See file : ‘6-way Splitter.xls’ for all split calculations)

7.1 Feed Network

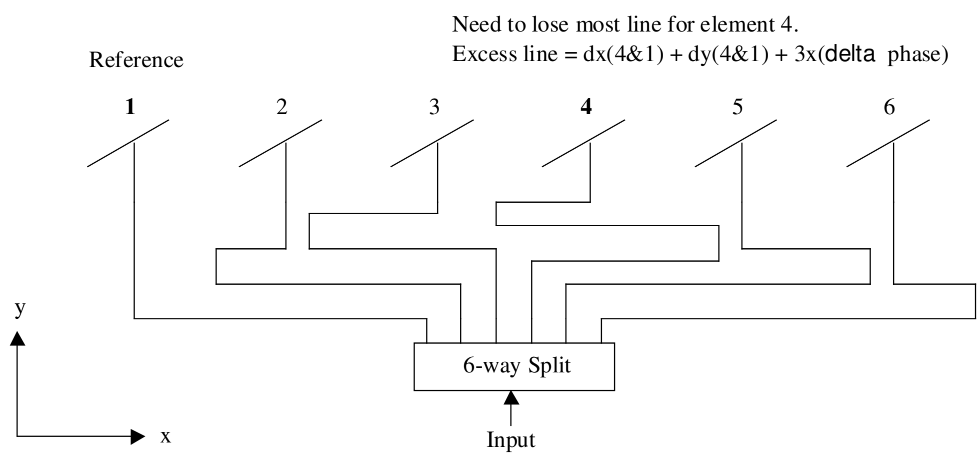
Having organised our array elements and power splitter topology, we need to turn our attention to getting power from the splitter to the elements with the correct phase. Depending on the transmission line technology used, this can be quite a challenge.

Generally speaking the best place to locate the power splitter is near to the physical centre of the array. This will minimise the total amount of transmission line required to reach all the elements, hence reduce the cost, loss and possible phase errors.

In doing this we find that the middle elements (3 & 4) in our array are the closest to the splitter, next (2 & 5) then (1 & 6). Now we also know that our array requires a successive phase delay of approx 60.6 Deg, from element to element (ref table 3.1-1).

The first step is to choose a reference element, ideally the one furthest from the splitter and with the least phase delay. Connection to this element defines the minimum length of transmission line we will require. Connection to all others will require the same amount of line plus the line to produce the incremental phase shift. With the caveat that if the required phase shift (∆phase) is greater than 360 Deg, then the phase shift can reduced to (∆phase-360 Deg). Hence the phases been given in modulo(360) by ArrayCalc [4].

In our case element 1 will be our reference element and define the minimum required transmission line. Additional line length due to the phase-delay and the fact that routes to the inner elements are shorter, will have to be 'lost' in the feed network. For our small array this is not a problem, the most line we have to 'lose' is for the connection to element 4, shown schematically in figure 7.1-1. For a large array 'losing' all this line can be a real problem, especially if your real estate is only 2D, as in microstrip.



Figue 7.1-1 Schematic of feed layout

There are of course numerous possibilities for implementing the feed network, microstrip, coax, stripline, waveguide or combinations of the above. Of these microstrip is probably the commonest for reasons of low cost and ease of manufacture. A few things to watch out for with this technology :

- Losses, even in low loss substrates these can be significant. E.g. Rogers RO4350 Er=3.43, thickness=0.76mm 35um Cu @ 2Ghz typical loss 1.8dB/meter. Compared to 0.2dB/meter for low loss coax.

- Coupling, avoid long lengths of line running parallel and close to each other. You may unintentionally be making an odd-even mode coupler. This can be particularly problematic in dual polarised systems, where cross-polar discrimination depends on good isolation between the feed networks.

- Resonances, because microstrip can potentially radiate it is normally shielded by a metal enclosure. Even if the walls of the enclosure have sufficient separation from the microstrip not to affect its impedance, the microstrip can couple into the cavity created and cause some interesting and unexpected results.

7.2 Modelling

In order to check out our design some form of modelling will be useful. The level of detail rather depends on what we hope to learn from it. Broadly there are 3 levels at which the modelling can be done.

1. Simple transmission lines: Where lines have parameters of impedance, phase length and possibly loss.

2. Piecewise modelling: Where transmission lines, junctions, bends and impedance-steps are represented by specific models, appropriate to the technology being used. E.g. Microstrip or Stripline components in ADS, QUCS etc.

3. Full-wave EM modelling: Where the whole physical structure is modelled. E.g. HFSS, IE3D, Sonnet etc

In an ideal world we would probably work our way though all three. The simple transmission line model can be used as a kind of continuity check, ensuring that all the basic calculations are correct. It can also be used to check for sensitivity to impedance variations due to tolerancing or mismatch errors at the outputs.

The piecewise modelling puts actual dimensions to our simple transmission line model, as well as adding another level of accuracy. Each of the junctions, bends and transmission lines take physical dimensions as their input parameters. Once a model in this form, there is enough information to proceed with a physical layout.

The initial physical layout may then be in the form of a full-wave EM model. The thing to remember here is that it is really like having a virtual breadboard model, it will only tell you what the performance would be if you built it. If it doesn't work you are still going to have to fix it. Some design modifications maybe easier and quicker on a virtual breadboard, others maybe easier and quicker on the real thing. If it is possible, there is nothing quite like getting your hands on some hardware and actually measuring it, to get a 'feel' for what you are doing – and it's fun.

Figure 7.2-1 below shows a simple transmission line model of the power splitter, including additional 50 ohm lines to achieve the required phase distribution. Figure 7.2-2 shows the results obtained, table 7.2-1 are the design values reproduced again for reference. Note that most measurement/modelling systems will output the phase in +/180 Deg format so be prepared to do some juggling.

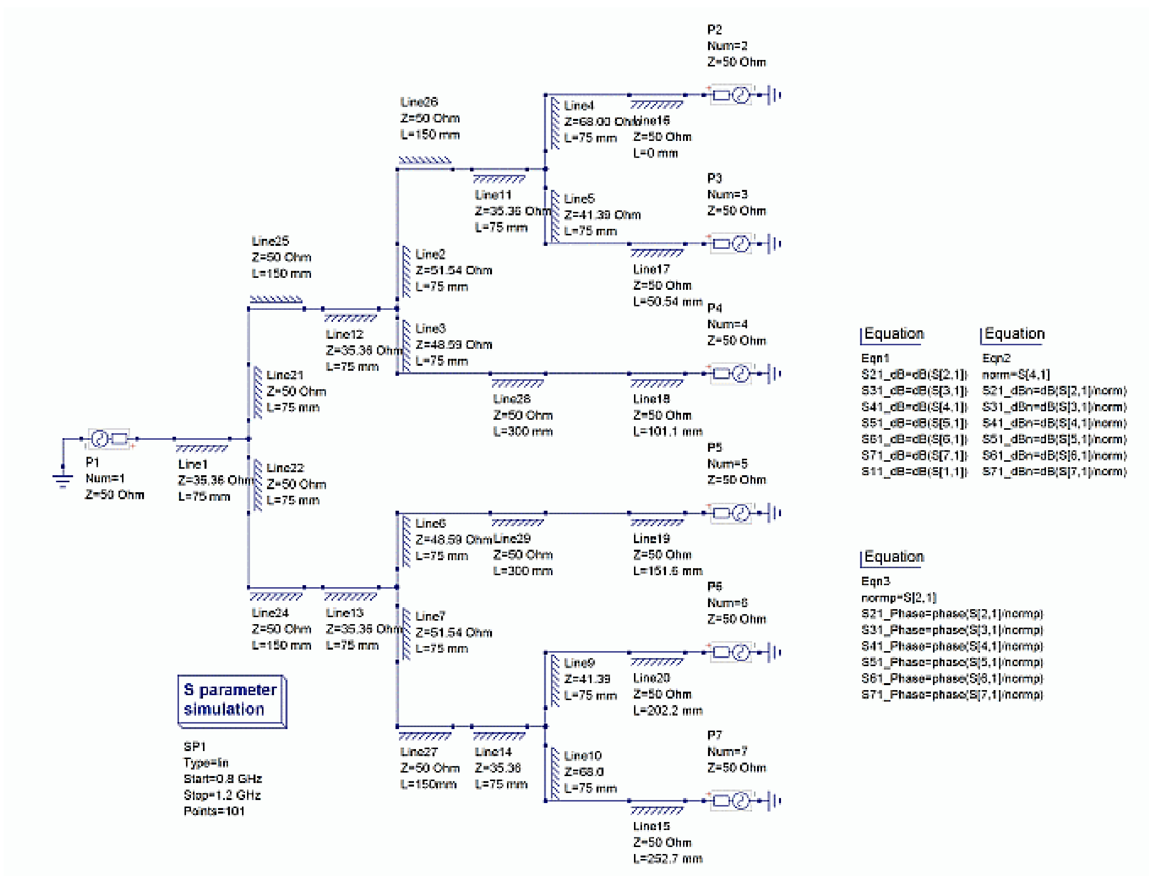


Figure 7.2-1 Transmission line model of splitter and phasing in QUCS

See QUCS file : '6-way2.sch'

The modelling was carried out using QUCS [6] and verifies quite nicely our calculations. Although the model is simple, the impedance values and lengths can be used in an appropriate transmission line calculator to produce the physical dimensions for a layout.

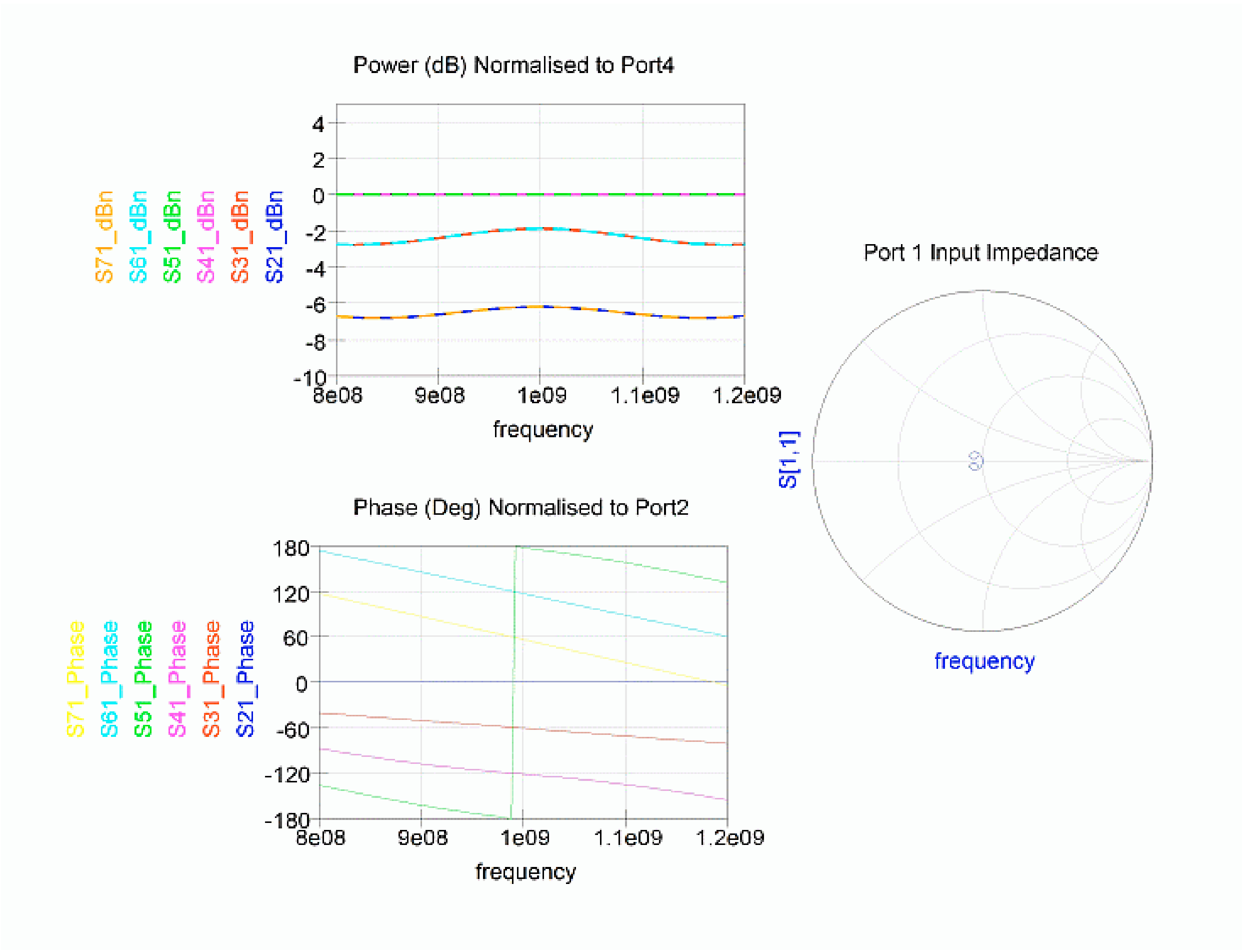


Figure 7.2-2 Results for the model in Figure 7.2-1

QUCS File : ‘6-way2.dpl’

Element No.	Ampl(dB)	Phase(0/360 Deg)	Phase(+/-180 Deg)
1	-6.19	0.00	-0.00
2	-1.88	60.65	-60.65
3	0	121.26	-121.26
4	0	181.82	+178.18
5	-1.88	242.34	+117.66
6	-6.19	302.82	+57.18

Table 7.2-2 Amplitude and Phase design values (Reproduced from Table 3.1-1)

8. THE COMPLETE NEC2 MODEL

Now we now have our array configuration (fig 5-1), active impedance matching (table 6.2-1) and power splitter design (fig 7.2-1), we can integrate them into a single model (fig 8-1), and see if it works.

The construction of this integrated model should be viewed as proof of concept rather than advice on 'this is how to do it'. As you will see if you look at the 4NEC2 input file 'Array 6-ele feed corr.nec', it is not an approach for the faint-hearted. There are now numerous simulation packages available that offer integrated EM / Circuit simulation in a more user friendly environment. Alternatively you could do it the old-fashioned way and just build a prototype.

As well as the grid wires, NEC2 also allows transmission lines to be included in the model as circuit theory type connections between wire segments. This means that the transmission lines are not represented physically in the model so can be connected arbitrarily. The only requirement is that lines are connected to a wire segment. If the wire is not to be part of the radiating structure it can be made much shorter than lambda, and just used as a connecting node. However to make the model 'intelligible' when viewed, some care needs taken when placing the node-wires, since the transmission lines are drawn as lines between the nodes. Figure 8-1 below shows the complete array model with finite ground plane and feed network. The peak directivity of 15.2dBi is in good agreement with the 15.3dBi given by ArrayCalc in the initial modelling in section 3.1.

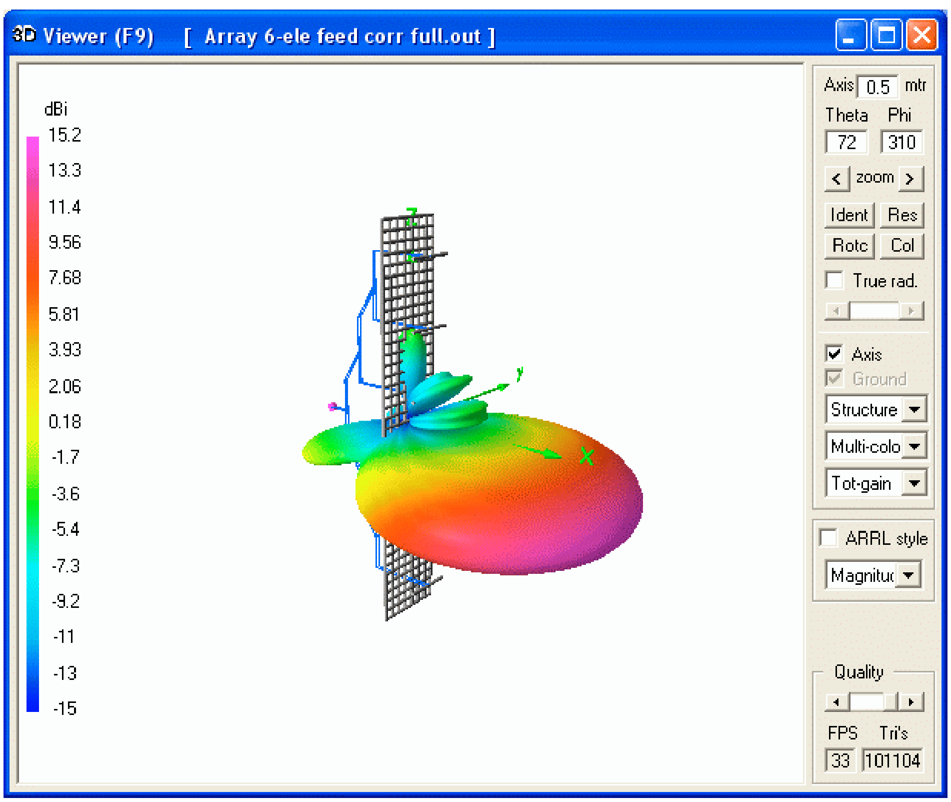


Figure 8-1 Full NEC2 model File : 'Array 6-ele feed corr.nec'

8.1 Patterns

To show the impact of various amounts of 'correction' the model was run 3 times. All runs used the same feed network and finite ground plane. The differences were in the dipole tuning and matching.

The first run represents what would happen if you used dipole elements tuned for free space. (by 'tuned' I mean resonant frequency and match)

The second run uses dipoles tuned for operation over the finite gridded ground plane.

The third uses dipoles tuned in the presence of the finite ground and all other elements excited in the correct amplitude and phase.

The corresponding theta patterns (phi=0 cut) are shown in Figure 8.1-1 below.

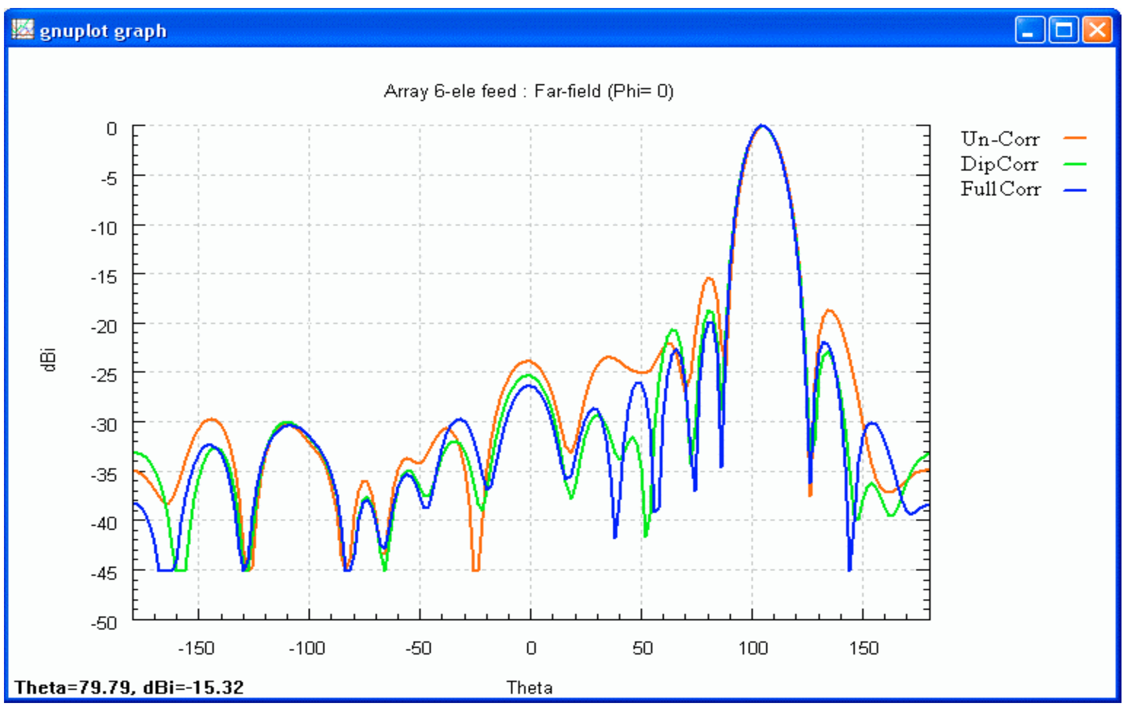


Figure 8.1-1 Normalised theta patterns for Phi=0 (1GHz).

Red : Uncorrected Free-space dipole parameters used, resonant length = 95% of (λ/2) and impedance assumed to be 73 Ohm +0j (impedance transformer = 60.4 Ohms) for all dipoles. The feed network is as designed together with phasing lines to produce 15deg down-tilt.

Green : Corrected dipole The dipole length has been adjusted for resonance in the presence of a ground-plane, resonant length=93% of (λ/2). The change in impedance to 80 Ohm +0j has also been corrected for (impedance transformer = 63.2 Ohms) for all dipoles. (ref section 4.1)

Blue : Fully Corrected The dipole length remains at 93% of (λ/2). Matching is now unique for each dipole using the active impedance transformers (Ref table 6.2-1).

The first thing to note about figure 8.1-1 is the substantial improvement in performance between the uncorrected and corrected-dipole plots. This is a useful result because it tells us that by simply tuning our array element in a physically representative environment (just adding a ground plane in this case), we can dramatically improve our chances of meeting the design target. Remember, the local environment for the element may also include a radome.

This element ‘tuning’ can be done in a modelled environment, as here (ref sections 4.1 & 5.1) or by constructing a physical prototype. If a prototype is to be used, but no feed-network is available, the presence of other elements can be accounted for to some extent by including them and terminating them with loads of characteristic impedance (typically 50 Ohms).

In terms of what a given element ‘sees’ as its local environment, there is often a distinction drawn between elements at the edge of the array and the inner elements. Generally speaking though precedence should be given to the inner elements, since in large arrays there are proportionately more of these. Also in most large arrays there will be some sort of amplitude taper, reducing the impact of mismatches from the edge elements.

Returning to figure 8-2 we can see that in all patterns in the 15deg tilt has been achieved, with the beam peak at theta=105deg (90+15deg from vertical Z-axis). However the fully-corrected pattern is the only one to meet side-lobe specification of of 20.0dB on the main beam, compared to 18.5dB for the corrected-dipole pattern.

Although the differences between the fully-corrected and corrected-dipole patterns are not huge for this example there are some points to bear in mind :

- The feed-network used had only the power split plus sufficient ‘phasing’ line to achieve the required phase distribution. In the actual array the lines will be longer because they must physically reach the element. The effect of un-corrected active element impedance would therefore be magnified.

- The ‘electrical tilt’ (scan angle) was only 15 deg, increasing this will accentuate the variation in active element impedances and therefore the amount of correction needed.

- All transmission lines are perfect in the model, in the actual array the impedances are unlikely to be exact, again magnifying the effect of mismatches.

Before finishing this section, it is worth looking at how the NEC2 model results compare to our original patterns calculated using ArrayCalc. The plot in figure 8.1-2 below shows the final ArrayCalc design from section 3.1 (red). Tuned dipoles over an infinite ground (green) and the final fully corrected NEC2 design (blue).

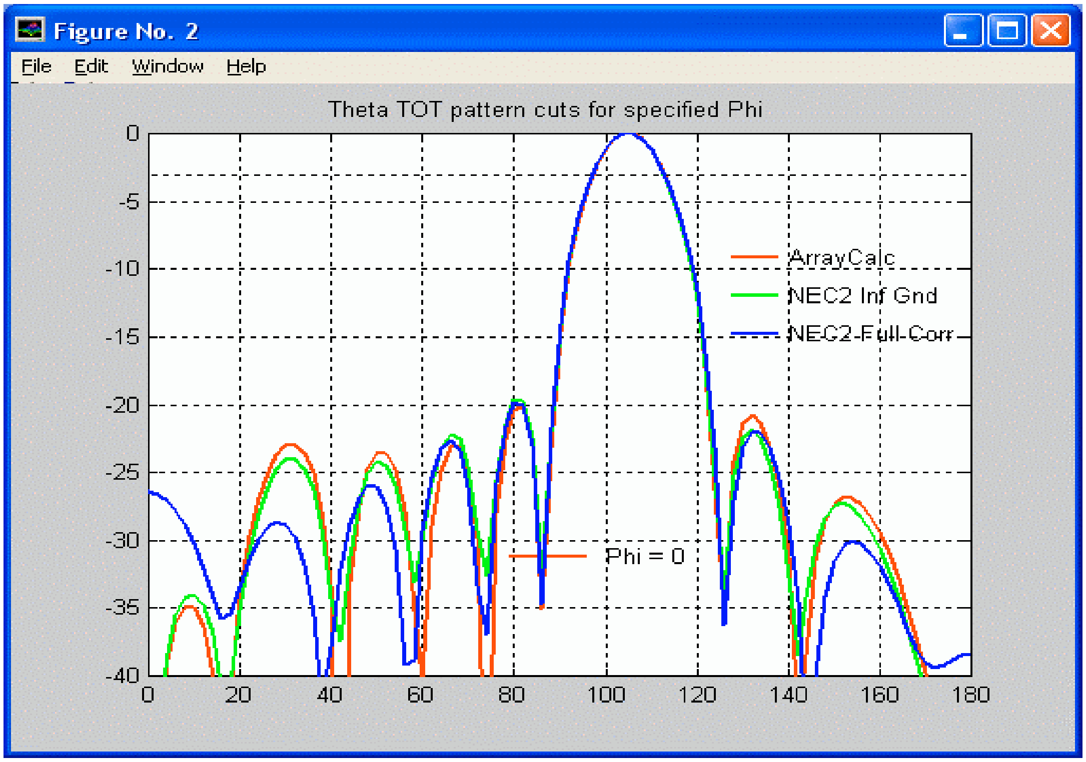


Figure 8.1-2 ArrayCalc / NEC2 comparisons

Red : ArrayCalc Array of 6 horizontal dipoles λ/2 at 1000MHz set 0.65λ apart and 0.25λ above the ground plane. Excitations as per table 3.1-1

Green : NEC2 Inf Gnd Array of 6 dipoles where the dipole length has been adjusted for resonance in the presence of a ground-plane, resonant length=93% of (λ/2). There is no feed network, each dipole has its own source excitation as per table 3.1-1

Blue : NEC2 Fully Corrected The dipole length remains at 93% of (λ/2). Groundplane is finite (0.7λ x 3.9λ) and represented by grid-wires. Feed network is included and matching is now unique for each dipole using the active impedance transformers (Ref table 6.2-1).

It can be seen from figure 8-3 that the 'ArrayCalc' and 'NEC2 Inf Gnd' are in good agreement. This shows that as long as some effort is made to account for the element's local environment (tuning the dipoles over the ground in this case), simple array-factor type software can give a good indication of array performance. The differences in the 'NEC2 Fully Corrected' trace are attributable mainly to the finite groundplane, whose effects will be more significant in a small array such as this. Even so, the main beam and 1st sidelobe levels are still show good agreement.

8.2 Impedance and Bandwidth

Achieving a good input impedance to the main input of a practical array can be a challenge, mainly due to the cumulative effects of all the manufacturing tolerances. In this example, with everything being 'ideal' the input impedance really just confirms that the sums were correct. However, the bandwidth of an array is usually determined by its ability to maintain the desired beam pattern over a frequency range, rather than its input impedance. The input impedance results for the full 4nec2 model are included here for completeness. Figure 8.2-1 shows the input impedance as Standing Wave Ratio (SWR), reflection coefficient and Smith Chart form.

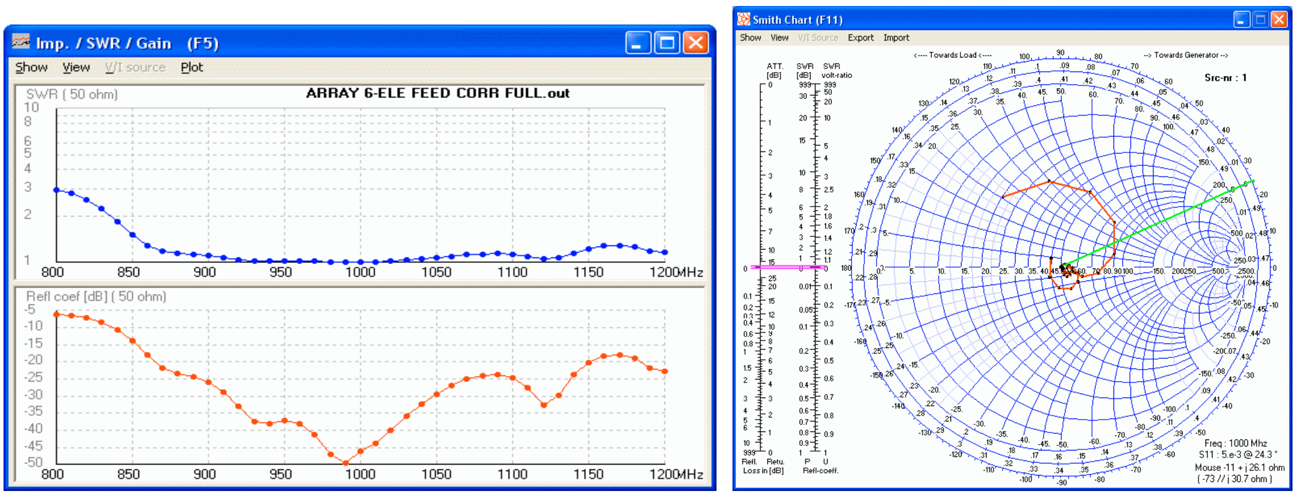


Figure 8.2-1 Full NEC2 model input impedance

If we now look at how the array patterns change with frequency, it is clear that pattern stability is likely to much more of an issue than input impedance, as far as bandwidth is concerned. The graph in figure 8.2-2 shows the patterns for 950 MHz,1000 MHz and 1050 Mhz (10% bandwidth).

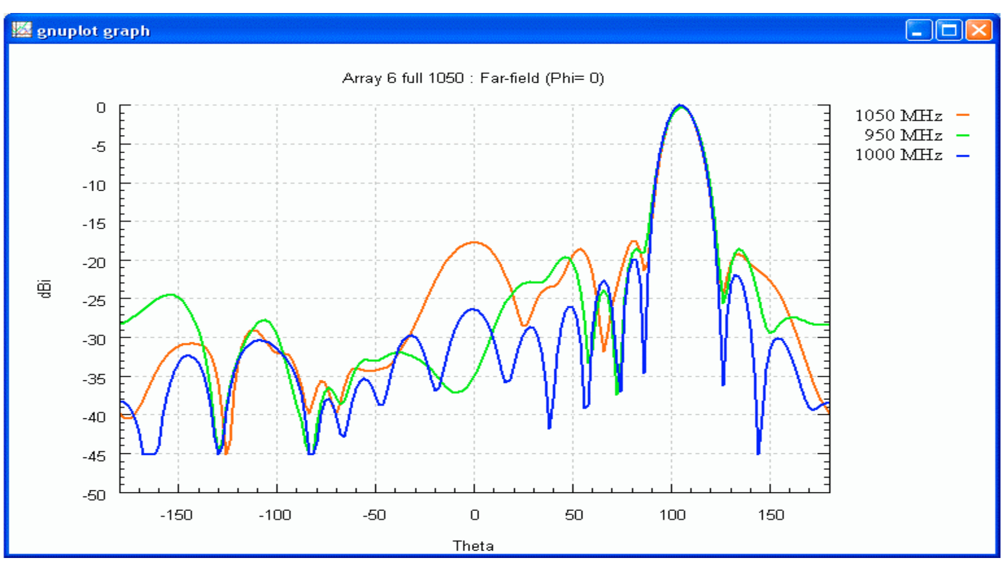


Figure 8.2-2 Pattern variation with frequency.

The patterns for 950 MHz and 1050 MHz show a rapid degradation in sidelobe performance either side of the design frequency. Notice what looks like a grating lobe has started to appear at 1050 MHz, at this frequency the element separation in terms of wavelength has increased from 0.65λ to 0.683λ. This is approaching the 0.7λ we had problems with in the initial design section 3.1. Ensuring that the element separation is correct for the maximum frequency of operation is particularly important, since a change in element spacing would require a lot of re-work. However the problems are also due to the fact that the array elements themselves are only matched exactly to the feed network at the centre frequency. The feed network in turn has ¼ wave transformers that only provide the exact impedance transformations at the centre frequency. As a result the actual excitation of the array can deviate significantly from the ideal, as you move away from the centre frequency.

In truth, the overall pattern stability with frequency depends on nearly all aspects of the array design. Techniques for broad-banding of the array elements include the use of parasitic elements, placing the element in a cavity or just using inherently broadband designs such as bow-tie dipoles or helices. The feed network can be broad-banded by using multiple ¼ wave steps or tapered lines.

The amount of 'phasing' line used to achieve the beam tilt is also important due to the cumulative error in the phase taper, as you move away from the centre frequency. This can sometimes be 're-distributed' with in the feed network to some advantage. Basically sections of 'phasing line' that would be common to a group of elements can be moved to any common section of the splitter. The overall phase length remains the same but the way the impedances are presented at the power splitter junction changes. I wouldn't recommend this unless you are fairly confident with all the other material, it can cause more problems than it solves.

In general it is important to try and mitigate as many of the predictable problems as possible, since there will be plenty of other small errors and deviations from the ideal that cause performance to slip away. An improvement of dB or two in the side-lobe levels may not seem a lot, but it might be the difference between meeting or failing the specification.

CONCLUSIONS

A 6-element horizontal dipole array, including feed network has been developed as a NEC2 model. In the process, aspects of basic array configuration, mutual coupling and feed network design have been examined.

It has been demonstrated that simple modelling techniques such as array-factor and transmission line analysis can provide a good basis for a workable design. More complex full-wave analysis has then been used to account for mutual coupling effects in the array, to get as close to the design goals as possible.

By breaking the problem down into manageable pieces, potential problems in each area have been identified and either solved or reduced. In doing this it is hoped that the reader has gained better understanding of how arrays work and how to design them.

REFERENCES

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[2] "Antennas" 2nd edition by John D. Kraus. Published McGraw-Hill ISBN 0-07-100482-3

[3] "Phased Array Design Toolbox for MATLAB, Theory of operation" by N. Tucker. ActiveFrance.com

[4] "ArrayCalc" Phased array design software for MATLAB, by N,Tucker. ActiveFrance.com

[5] "4NEC2" Pre and Post processing software for Numerical Electromagnetic Code (NEC2) by Arie Voors

[6] "QUCS" Quite Universal Circuit Simulator by Michael Margraf et al

APPENDIX A (APERTURE DISTRIBUTIONS)

Chebyshev Distribution

The article below is taken straight from dsprelated.com, simply because I think it is the best and most informative description of how to compute the Chebychev window coefficients I have found. It is also the algorithm I have used for the matlab file chebwin1.m. It gives exactly the same results as MATLAB's standard chebwin.m, but I feel the algorithm is easier to follow.

Notice the role of the Discrete Fourier Transform (DFT) in computing the required aperture excitation. This, like many beam synthesis methods actually starts with a function or set of data points representing the desired far-field pattern. In this case the data is obtained using a summation of Chebychev polynomials, which is then Inverse DFT'd to find the required element excitations.

Computing Chebyshev Window Sequences
Posted by Rick Lyons on Jan 7 2008 under Academia / Research (www.dsprelated.com)

Chebyshev windows (also called Dolph-Chebyshev, or Tchebyschev windows), have several useful properties. Those windows, unlike the fixed Hanning, Hamming, or Blackman window functions, have adjustable sidelobe levels. For a given user-defined sidelobe level and window sequence length, Chebyshev windows yield the most narrow mainlobe compared to any fixed window functions.

However, for some reason, detailed descriptions of how to compute Chebyshev window sequences are not readily available in the standard DSP textbooks, and I'm not rightly sure why that is. In any case, here are the steps for one method of computing M-sample symmetrical Chebyshev window sequences:

1. If the desired Chebyshev window sequence length is M, then define integer N = M-1.

2. Define the window's sidelobe-level control parameter as γ ,the window's sidelobe levels, relative to the mainlobe peak. (For example, if we desire frequency-domain maximum sidelobe levels of 60 dB below the mainlobe's peak magnitude then we set $\gamma = 60$.)

3. Compute parameter α as $\alpha = \cosh[\cosh^{-1}(10^{\gamma/20})/N]$.

4. Compute the N-length sequence A(m) using $A(m) = |\alpha \bullet \cos(\pi m/N)|$ where the index m is $0 \leq m \leq (N-1)$.

5. For each m, evaluate the Nth-degree Chebyshev polynomial whose argument is A(m) to generate a frequency-domain sequence W(m). There are many ways to evaluate Chebyshev polynomials. Due to its simplicity of notation, I suggest the following:

   $W(m) = (-1)m \bullet \cosh\{N \bullet \cosh^{-1}[A(m)]\}$, when $|A(m)| > 1$,

   or

   $W(m) = (-1)m \bullet \cos\{N \bullet \cos^{-1}[A(m)]\}$, when $|A(m)| \leq 1$.

   The W(m) sequence is real-only, although our software's computational numerical errors may produce a complex-valued W(m) with very small imaginary parts. Those imaginary parts, if they exist, should be ignored. The above (-1)m factors are necessary because the frequency-domain index m is never less than zero.

6. Compute a preliminary time-domain window sequence, w(m), using w(m) = real part of the N-point inverse DFT of W(m).

7. Replace w(0), the first w(m) time sample, with w(0)/2.

8. Append that new w(0) sample value to the end of the N-point w(m) sequence, w(M-1) = w(0), creating the desired M-length window sequence w(k) where time index k is $0 \leq k \leq (M-1)$.
9. Normalize the amplitude of w(k), to obtain a unity peak amplitude, by dividing each sample of w(k) from Step 8 by the maximum sample value in w(k).

The above procedure seems a bit involved but it's not really so bad, as the following Chebyshev window design example will show. Assume we need a 9-sample Chebyshev window function whose frequency-domain sidelobes are -60 dB below the window's mainlobe level. Given those requirements; M = 9, N = 8, $\gamma = 60$, and

$a = \cosh[\cosh^{-1}(10^{60/20})/8] = 1.4863$.

After the inverse DFT operation in the above Step 6, w(m=0)/2 = 11.91, thus we set w(k=0) = w(k=8) = 11.91. The maximum value of w(k) is 229.6323, so we divide w(k) by that value yielding our final normalized 9-sample Chebyshev window sequence listed in the rightmost column of Table 1.

Table 1 Nine-point Chebyshev window computations

m	*A*(*m*)	*W*(*m*)	*w*(*m*)	*k*	*w*(*k*)	*w*(*k*) norm.
0	1.4863	1000.00	23.8214	0	11.910	0.0519
1	1.3732	-411.49	52.1550	1	52.1550	0.2271
2	1.0510	6.41	123.5232	2	123.5232	0.5379
3	0.5688	-0.13	197.5950	3	197.5950	0.8605
4	0.0000	1.00	229.6323	4	229.6323	1.0000
5	-0.5688	-0.13	197.5950	5	197.5950	0.8605
6	-1.0510	6.41	123.5232	6	123.5232	0.5379
7	-1.3732	-411.49	52.1550	7	52.1550	0.2271
				8	11.910	0.0519

Acknowledgement: I thank DSP guru fredric j. harris (San Diego State Univ.) for his generous personal guidance in helping me create this procedure.

Further Reading: I ran across an interesting website that discusses evaluating Chebyshev polynomials, and computing Chebyshev window sequences in conjunction with designing FIR filters. That website is the paper: "Notes of the Design of Optimal FIR Filters" by John R. Treichler at: http://www.appsig.com/products/tn070.htm

Modified Taylor Distribution

This distribution was also derived by starting with the desired far-field pattern, in this case a 'modified' sin(x)/x type pattern Eq A-1, which is where the distribution gets its name. Since the far-field pattern is a continuous function, rather than a summation, a standard Inverse Finite Fourier Transform could be used (rather than discrete) resulting in a continuous aperture distribution function Eq A-3.

$$E(u) = \frac{\sin\sqrt{u^2 - \pi^2 B^2}}{\sqrt{u^2 - \pi^2 B^2}} \qquad \text{Eq A-1}$$

Where

$u = (\pi \cdot l / \lambda)\sin\phi$

l = Overall length of aperture

ϕ = Angle from bore-sight

Relation of Taylor parameter B to sidelobe level ratio R is given by

$$R = 4.603 \cdot \frac{\sinh(\pi \cdot B)}{\pi \cdot B} \qquad \text{Eq A-2}$$

Where :

R = Linear ratio of 1st side-lobe level w.r.t. main beam i.e. $R=10^{R(dB)/20}$

B = Taylor function parameter, when B=0 the function represents a standard uniform aperture with no side-lobe suppression and R=4.603 (13.2dB).

The symmetrical aperture distribution to produce the suppressed side-lobe pattern is given by

$$f(x) = J_o\left(j \cdot \pi \cdot B\sqrt{1 - x^2}\right) \qquad \text{Eq A-3}$$

Where :

J_o= Zero order Bessel function of the first kind.

x = is the position within a 2 unit length aperture i.e. $-1 < x < 1$

Some values of B to give specific values of R are given in Table-A1 below

SL (dB)	B
13.26	0
15	0.3558
20	0.7386
25	1.0229
30	1.2762
35	1.5136
40	1.7415
45	1.9628
50	2.1793

Table-A1

Or B can be evaluated as a function of R(dB) using the following 6^{th} order curve fit, as used in the MATLAB files ModTaylor.m and taywin_array.m in ArrayCalc:

Cp6 = -8.6865e-10;

Cp5 = +2.3255183e-7;

Cp4 = -2.519124552e-5;

Cp3 = +1.41192273253e-3;

Cp2 = -4.329667260471e-2;

Cp1 = +7.3879146026700e-1;

Cp0 = -4.66189278748759e-0;

B=Cp6*(R^6)+Cp5*(R^5)+Cp4*(R^4)+Cp3*(R^3)+Cp2*(R^2)+Cp1*R+Cp0;